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A note on division orderings on strings.

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An ordering is an irreflexive, transitive relation; it is total if any two elements are comparable. Let A be a finite alphabet, A^* the set of words over A, and ε the empty word. A division ordering on A^* is an ordering $>$ such that for all nonempty x, y, u, and v in $A^* - \{\varepsilon\}$ we have $x > \varepsilon$, and if $u > v$ then $xuy > xvy$.

A total division ordering is called tame if, given a total ordering of A^*, there is a relabelling of the elements of A as $a_1, \cdots, a_n$ such that for each $w \in A^*$, $G(w) = a_1^{\alpha_1} \cdots a_n^{\alpha_n}$ and $S(w) = a_n^{\alpha_n} \cdots a_1^{\alpha_1}$; here $G(w)$ and $S(w)$ are the largest and smallest permutations of w, respectively, and $\alpha_i = n(a_i, w)$, i.e., the number of occurrences of a_i in w.

The author shows that for words v and w over A, and a total division ordering $>$ on A^*, (1) if $>$ is tame, and $n(a_i, v) < n(a_i, w)$ for each i, then $v < w$; (2) if $A = \{a, b\}$, $n(a, v) < n(a, w)$, and $n(b, \nu) < n(b, w)$, then $v < w$; (3) not all division orderings are tame.

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