On the equivalence and containment problems for unambiguous regular expressions, regular grammars and finite automata.


The PSPACE-completeness of the equivalence and containment problems for regular language descriptors (i.e., regular expressions, regular grammars and finite automata) has been established by L. J. Stockmeyer and A. R. Meyer [Fifth annual ACM symposium on theory of computing (Austin, Tex., 1973), 1–9, ACM, New York, 1973; MR0418518 (54 #6557)]. In the paper under review the authors consider these problems for unambiguous regular language descriptors as well as for the case in which the ambiguity is bounded by $k$. A language descriptor is unambiguous [resp., its ambiguity is bounded by $k$] if each word in the corresponding language can be generated or accepted in only one way [resp., in at most $k$ ways]. The main results show that for unambiguous descriptors and for descriptors of bounded ambiguity these problems are solvable deterministically in polynomial time. (Therefore the results of Stockmeyer and Meyer depend on highly ambiguous regular language descriptors in an inherent way.) The proofs consist of deterministic polynomial-time algorithms based on some elementary properties of systems of homogeneous linear difference equations with constant coefficients that are related to the number of accepting computations of strings of length $n$ by a nondeterministic finite automaton.

Some evidence is given to show that these results can hardly be improved; e.g., it is shown that the equivalence and containment problems for regular language descriptors of bounded degree of ambiguity (i.e., their ambiguity is bounded by some $k$) are CoNP-hard.

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