Cross-validation of 3D particle tracking velocimetry for the study of granular flows down rotating chutes

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HIGHLIGHTS

• A 3D particle tracking technique is cross-validated against independent methods.
• Granular particles are flowing down a rotating chute.
• Surface particle velocities and particle bed height are obtained simultaneously.
• Discrete Element Model simulations are validated.

GRAPHICAL ABSTRACT

ABSTRACT

Three-dimensional particle tracking velocimetry (3D-PTV) is a promising technique to study the behavior of granular flows. The aim of this paper is to cross-validate 3D-PTV against independent or more established techniques, such as particle image velocimetry (PIV), electronic ultrasonic sensor measurements for bed height, and the discrete element model (DEM), for the complicating circumstance in which granular particles are flowing down a rotating chute. 3D-PTV was used to gain access to Lagrangian trajectory data of surface particles in such flows, from which independent measurements of both the surface velocity and the bed height in the chute were derived. The 3D-PTV method was based on imaging and tracking colored tracer particles that were introduced in the granular material, which are viewed from three directions. The three cameras collected consecutive frames a known time interval apart and the PTV algorithm for locating and tracking particles was used to determine particle trajectories and velocities. We found that the 3D-PTV results are in good agreement with PIV results with regard to the streamwise and spanwise surface velocity of particles in the rotating chute. The particle bed height obtained from 3D-PTV was also found to be in good agreement with data from an ultrasonic bed-height sensor. The experimental findings from PTV for the non-rotating case were used to tune the friction coefficient in our DEM simulations. The simulation method was validated by the good agreement between experimental findings and simulations at all rotation rates studied.

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1. Introduction

Experimental studies of granular flows are difficult to perform due to the opaque nature of such materials. Nevertheless, insight has been generated through use of magnetic resonance imaging...
(Kawaguchi, 2010; Hill et al., 1997), digital imaging (Guler et al., 1999; Capart et al., 2002; Bonamy et al., 2002), particle image velocimetry (PIV) applied to 2D granular flows (Rokkers et al., 2004; Laverman et al., 2008; Zeilstra et al., 2008) and particle tracking velocimetry (PTV) (Chou and Lee, 2009; Yang and Hsiau, 2006; Liao and Hsiau, 2009; Jasti and Higgs, 2008; Veje et al., 1999; Jain et al., 2002). In the past a large amount of experimental work has been performed on granular flows through inclined channels, see e.g. Augenstein and Hogg (1978), Brennen et al. (1983), Campbell and Brennen (1985), Johnson et al. (1990), Holyoake (2011), Ottino and Khakhar (2000), Khakhar et al. (1999), Savage and Lun (1988), Pouliquen and Renaut (1996), Silbert (2001), Ancey (2001), Barbolini et al. (2005), and Baran et al. (2006).

Most of the above experimental studies are focused on fixed chutes. However, in some applications such as in the operation of blast furnaces in the metallurgical industry, the chutes are rotating. The chutes may even rotate at such high rates that Coriolis and centrifugal forces start to play a significant role, leading to flows and particle distributions deviating considerably from those in non-rotating chutes. Reliable measurements are essential to obtain a detailed understanding of such granular flows in rotating chutes. Three-dimensional particle tracking velocimetry is a promising technique, because it can potentially deliver with high spatial and temporal resolution the particle bed height and 3D velocity profile. The bed height can also be obtained by using ultrasonic height sensors, but the resolution of such sensors is lower, while the experimental procedure is much more labour intensive or expensive (a new experiment needs to be performed for each measurement position, or a large number of sensors needs to be used). The velocity profile can also be obtained by using particle image velocimetry, but usually only in two dimensions.

In this paper we will focus on cross-validation of the 3D-PTV technique, for applications of granular flows in chutes rotating at significant rotation rates. Note that by ‘cross-validation’ we do not mean the statistical technique of estimating the performance of a predictive model. Rather we mean the practice of confirming experimental findings from one technique by repeating the experiments using independent other techniques.

The 3D-PTV technique is a flexible non-intrusive image analysis technique for flow measurements. It was first introduced by Chang et al. (1984) and was further developed by Racca and Dewey (1988). This technique has a history of development for more than a decade at the Institute of Geodesy and Photogrammetry (IGP) and at the Institute for Hydrodynamics and Water Resources Management (IHW) both at the Swiss Federal Institute (ETH) Zurich (Maas, 1991; Malik et al., 1993; Maas, 1995; Maas et al., 2002). In a previous paper we used the PIV technique to measure the surface particle velocity and an electronic ultrasonic sensor to measure the bed height in a rotating chute (Shirsath, 2013). In contrast to PIV, PTV is able to track individual particles in the flow and provides both the Lagrangian and the Eulerian representation of the flow field (Willneff, 2002). To be able to track individual particles instead of providing a global (Eulerian) flow field, PTV requires three cameras to detect the position of the particle in the three dimensional domain. The cameras capture images of the flow from different angles. From these images, it is possible to determine the position of an individual particle and compute its trajectory. In PTV it is necessary to make a clear distinction between the particles which are actually tracked and all other particles. Therefore, tracer particles are introduced in the granular flow. Obviously, the concentration of tracer particles may not be too high, otherwise it becomes difficult to distinguish individual tracer particles, and the method becomes less accurate (Prasad, 2000). We perform our analysis on a system of monodisperse 3 mm spherical glass particles, flowing down a rectangular plexiglass chute inclined at 30° and rotating at various rotation rates. The mass rate used is 3.2 kg/s. In our previous paper (Shirsath, 2014), we published a discrete element method (DEM) validation using experimental results of surface velocity obtained by PIV and bed height from an ultrasonic sensor, using a lower mass rate of 1.6 kg/s.

Besides the primary goal of assessing the ability of the 3D-PTV technique to provide surface information such as surface velocity of particle and particle bed height, as a secondary goal we will use the results of this work to further validate our DEM simulations (Shirsath, 2014). Recently, a number of papers focusing on DEM simulations of granular flows in the blast furnace charging process have appeared (Mio et al., 2008, 2009, 2010, 2012; Ho et al., 2009; Zhou et al., 2011; Yu and Saxén, 2010, 2011; Yu, 2013; Yu and Saxén, 2013; Wu et al., 2013; Bhattacharya and McCarthy, 2014; Akashi et al., 2008; Zhang et al., 2014). Given its popularity, it is crucial that the DEM method is properly validated against precise experiments, including cases in which the process equipment is rotating.

The paper is organized as follows. In Section 2 we present the experimental set-up and its procedure. In Section 3, the measurement techniques using PIV, PTV and the electronic ultrasonic sensor are presented. In Section 4, the numerical model is explained briefly. In Section 5, we present our post-processing methods to obtain data from the numerical model which are similar in spirit to the experimental measurements. In Section 6, we present the experimental results, including a repeatability study of the PTV experiments for bed height and streamwise surface particle velocity. We compare the bed height from PTV with independent measurements using an ultrasonic height sensor, and compare the surface velocity from PTV with independent measurements using PIV. In Section 7, we validate our DEM simulations by comparing with the experimental findings of bed height, streamwise velocity and spanwise velocity for different rotation rates of the chute. We end with our conclusions.

2. Experimental setup and procedure

In this section we describe the hardware of the 3D-PTV system, consisting of the rotating table facility, camera system and illumination facility.

2.1. Experimental setup

The experimental equipment includes a plexiglass chute, a hopper for storage of particles, and a collection tank. A dynamic weighing scale was used to measure the mass rate, and PTV and an electronic ultrasonic sensor were used to measure surface information such as particle bed height and the surface particle velocity field.

The granular particles were deposited onto a rectangular plexiglass chute straight from the hopper mouth. The bottom wall of the chute was white to achieve a better contrast in the photographs for detecting tracer particles. The whole chute was fixed at its bottom to a strong metal plate of 1 cm thickness to minimize vibrations caused by the particles poured from the hopper mouth to the chute surface at the inlet. The inclination angle of the chute was adjustable between 0 and 70° with respect to the horizontal. At the end of the chute, a collection tank received the granular particles. This tank was placed on a dynamic weighing scale to measure the exit mass flow rate.

The whole setup was mounted on a rotating table, so that the flow was measured in the non-inertial frame of reference. All the hardware, constituting the focusing system and the measurement system, was located on the top of the table or surface below it as
shown in Fig. 1. The equipment was remotely controlled from an adjacent room for safety precautions during the rotating experiments. Details of the rotating table and camera system are described in the next subsections.

2.1.1. Rotating table
The rotating table at the Fluid Dynamics Laboratory (Department of Applied Physics, Eindhoven University of Technology) used in this research was designed to support heavy constructions up to a weight of 1000 kg. Furthermore, the table has the ability to spin at a constant angular velocity in the range of 0.01–10 rad/s, with an accuracy of 0.005 rad. del Castello and Clercx (2013) and van Bokhoven et al. (2009) employed this rotating table facility for rotating turbulence studies. They did several experiments and tests to determine the exact stability and accuracy of the table.

In our experiments we chose a particular range of rotation rates between 0 and 24 rotations per minute (0 and 2.51 rad/s) because we expected a transition in the behavior occurring within this range. The dimensionless Rossby number and Froude numbers for the experimental setup, based on the rotation rates and maximum particle velocity at the end of chute, are given in Table 1. Since the PIV and PTV measurements are highly sensitive for camera movements during the experiment, a stable rotational operation was absolutely necessary.

2.1.2. Camera system
Three high speed cameras were situated above the chute, used to capture the images that were used for the optical measurement techniques particle image velocimetry (PIV) and particle tracking velocimetry (PTV). PIV was used to measure the surface velocity of the granular flow, whereas PTV was used to measure the trajectories of the individual particles along the length of the chute. For PIV only one camera was needed to capture images of the granular flow. It was positioned perpendicular above the center of the chute at a distance of 1.6 m (the middle camera). For PTV three cameras are needed, one of which has the same position as for PIV, while the two other cameras were placed at an up and downstream position, respectively, and were slightly tilted, see Fig. 1. From the three different angles the position of particles can be determined and subsequently the trajectory of individual particles. A Nikon 28 mm f/2.8 lens was used to cover the entire length of the chute for all three cameras. The image magnification was 1:55, i.e. each

<table>
<thead>
<tr>
<th>Rotation rate (rpm)</th>
<th>Rossby number</th>
<th>Froude number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ω = 8</td>
<td>1.879</td>
<td>0.055</td>
</tr>
<tr>
<td>Ω = 16</td>
<td>0.939</td>
<td>0.218</td>
</tr>
<tr>
<td>Ω = 24</td>
<td>0.627</td>
<td>0.491</td>
</tr>
</tbody>
</table>
3 mm particle was imaged to a size of 55 μm on the camera sensor. Given the pixel size of 17 μm, this corresponds to 3.2 pixels for a particle. The three cameras were synchronized so that images were captured at the same time from all cameras. The high speed cameras (Photron FastcamX-1024PCI) have an internal memory of 12 GB RAM to save up to 60,000 images, which is equal to a saving time of roughly 30 seconds when using a frame rate of 2000 frames per second and a resolution of 128 × 1024 pixels. This saving time of 30 s is sufficient to execute a single experiment, since the maximum experimental time was 9 seconds (total weight granular material was 30 kg; flow rate was 3.2 kg/s). To be able to save the captured images and to run the PIV/PTV software, a computer system was needed. After each experiment, the images were transferred from the internal memory of the cameras to this computer system. From there on, the images can be processed using the specific PIV and PTV software.

2.1.3. Illumination system

The high frame rate of the cameras implies a very short exposure time. Therefore, the illumination system had to be strong and homogeneous enough for the cameras to see the light reflected by the granular particles, including the tracer particles, in every part of the measurement field. The LED light was supplied by Ledlight Group, Germany, and is using a LED light power supply of 95 W max (V\text{max}: 30–350 V DC, I\text{eff}: 350 mA-constant current, power factor correction (PFC): 0.98.) Four LED lights were used to illuminate the whole chute. Each LED Light contains 72 small blue spherical glass spheres, which have the same collision properties of these particles should be close to the properties of PTV it has to be seeded with tracer particles. The physical flow behavior. In our experiments we used 3 mm blue spherical glass spheres, which have the same collision properties as the other transparent glass particles. We mixed 1 kg of these tracer particles into 30 kg of glass particles.

2.1.4. Tracer particles

To properly visualize the granular in the rotating chute for 3D-PTV it has to be seeded with tracer particles. The physical properties of these particles should be close to the properties of the flowing granular particles to guarantee that they properly represent the flow behavior. In our experiments we used 3 mm blue spherical glass spheres, which have the same collision properties as the other transparent glass particles. We mixed 1 kg of these tracer particles into 30 kg of glass particles.

2.1.5. Calibration plate

In Fig. 2 (bottom figure), the top view of the calibration plate used for the experiments is shown. The calibration plate had a thickness of 0.009 m and was 0.08 m wide and 1 m long, which was sufficient to cover the entire area of the chute. The calibration plate contained 5 × 62 calibration points. The calibration points had a diameter of 4 mm and the distance from the center of one calibration point to the next was 16 mm. Furthermore, three calibration points had a larger diameter, to identify the center and orientation of the calibration plate during processing of the calibration phase.

2.2. Experimental procedure

During the start-up of the experiments, the following steps were performed. First, the particles were filled in the hopper. Then the rotating table was accelerated until it reached a steady rotation rate. Then an experiment was started by opening the outlet of the hopper by remote control. Measurements were made continuously until the hopper was depleted. The time range during which steady-state flow conditions apply was determined after the experiment by analyzing the time dependence of mass, height and velocity measurements.

![Fig. 2. Raw images of granular flow in a chute rotating at 24 rpm, captured by 3 PTV cameras at the same time instant. The group of tracer particles circled in the three images represents the same particles as seen by the different cameras. The bottom figure shows the top view of the calibration plate used in the experiments.](image-url)
to determine the 3D position in the object space by forward intersection. Using epipolar constraints, homologous image points can be detected and the 3D object coordinate of each particle can be determined. Finally, temporal matching is performed to determine trajectories of each tracer particle.

With the present setup, up to 400 particles per time step were tracked on average. This number is not a hard limit of the method. In a larger setup, e.g. when using a wider chute, a much larger number of tracer particles could have been tracked. The limitation in this number lies entirely in the requirement of having a sufficiently low seeding density to ensure a large separation (relative to the diameter) between the visible tracer particles.

Details of the algorithms used in our code can be found in the literature: for calibration and 3D positioning algorithms we refer to Maas et al. (1993); for the temporal tracking algorithm to Malik et al. (1993); and for the latest developments of the tracking routine to Willneff (2002, 2003), Willneff et al. (2012). Below we will discuss the most important steps.

3.3.1. Calibration of the camera system

Calibration of the camera means determining the extrinsic and intrinsic parameters using a set of image points with known world coordinates. A multi-planar calibration method is used, in which multiple images are captured of the same planar calibration plate at different heights within the experimental volume. The accuracy of the calibration is highly dependent on the correct positioning of the plate (Castello, 2010). For this reason, special care has been taken to accurately position the calibration plate throughout the measurement volume, to minimize the disadvantage of the multi-planar calibration method. The larger the volume covered by the calibration plate, the smaller are interpolation and extrapolation errors of the calibration functions (Castello, 2010).

To create the calibration functions which correlate the pixel information of each camera to the world coordinates, recordings of the calibration unit have been carried out with the same orientation: the calibration plate plane is parallel to the chute base. Translation in the coordinate direction perpendicular to the calibration plate plane is carried out. The recordings of the calibration plate are registered in 4 different positions, at a height of 9 mm, 19 mm, 34 mm and 54 mm, with typical positioning error of less than 0.5 mm. With the information of the pixel size of the camera sensors, and the diameter of the circular voids, 3rd order polynomials relate the pixel information to the physical dimensions of the calibration plate. Linear interpolations and extrapolations of the generated polynomials are extended to the whole measurement volume.

3.3.2. Three-dimensional positioning of the particles

The second important phase of 3D-PTV processing is the three-dimensional positioning of particles in space from the raw images. Fig. 2 shows an example of the raw images simultaneously generated by the three cameras.

Several processing steps are required to obtain the particle position from the captured images. The first two steps are preprocessing steps, namely high pass filtering, and particle detection and location of the particles. High pass filtering is necessary to remove non-uniformities in the background illumination (Willneff, 2002). For the detected and located particles in the preprocessing stage, the corresponding particles for all cameras are established. Since the corresponding particles are detected by all cameras, subsequently the 3D coordinates of these particles can be determined with the obtained calibration data from the first phase. The resulting error in the particle position is 0.5 mm in the horizontal plane and 1.0 mm in the depth direction.

From the 3D coordinates, the particles can be tracked in the object and image space.

3.3.3. Particle trajectory reconstruction

After obtaining the 3D coordinates of each particle, a particle trajectory is constructed by comparing the positions of particles in consecutive image frames. For each frame, the software tries to find an individual particle based on the position of that particular particle in the previous frame(s). Later the displacement of an individual particle between two frames (a single time step) is evaluated. In order to find such a displacement, at least two cameras are needed. The trajectory of a particle that is detected by an individual camera can be imaged as a two-dimensional path, also called epipolar lines. If a second camera detects the same particle trajectory, a three-dimensional path can be determined from both two-dimensional pathways. In PTV practice, three or even four cameras are used. The redundant information is used to improve the accuracy of the method. In this research, only trajectories of particles which are detected by all three cameras were used for further analysis. Furthermore, because particle trajectories with just a few number of positions have a higher probability of being false than trajectories with a large number of positions, we only used trajectories with a minimum number of positions. This procedure has proven very useful to remove unrealistic particle trajectories. Elimination of particle trajectories comprising less than 10 spatial positions has been found to be adequate in our work.

4. Simulation model

4.1. Discrete element model

The results of this work will be used to further validate the discrete element method (DEM) presented in our previous work (Shirsath, 2014). DEM can be used to parametrically investigate the role of particle interactions on the granular flow, as well as to gain access to properties which are not readily available from experiments. The DEM used in our work is based on the linear spring/dashpot soft-sphere model, originally developed by Cundall and Strack (1979).

In our previous work (Shirsath, 2014) we introduced the particles in a rectangular area with a bcc-lattice arrangement of the particles, parallel to the chute bottom wall, where the initial velocity is set by the required mass rate. We then found that the resulting bed height and particle velocity profiles were independent of the exact manner of introducing the particles after a distance of approximately 0.2 m from the inlet. To reproduce the experiments somewhat more closely, in this work we introduce the particles parallel to gravity, but still using the bcc-lattice arrangement for convenience. Fig. 3 shows a snapshot of introducing particles at the inlet in this manner. We determined the first length wise position at which the bed height and surface flow velocity match between experiment and simulation for a non-rotating chute at an angle of inclination of 30°, and subsequently placed the axis of rotation in our simulations at the same distance from this location as in the experiments to ensure the same centrifugal forces are felt by the particles.

4.2. Simulation settings

The physical properties of the spherical glass particles, the chute dimensions, and the flow conditions in our DEM simulations are kept the same as in our lab-scale experiments, as shown in Table 2. In the simulation, the chute is initially empty, as is the case in the physical experiments. Simulations are carried out for a
constant mass flow rate at the inlet of the chute. We introduce the particles with a mass flow rate of 3.2 kg/s, which is equal to the mass flow rate of the experiments. With this mass flow rate, the total number of particles in the chute is typically 55,000.

The simulations were carried out on a single core of an Intel Xeon E5520 processor (at 2.27 GHz). In general each simulation required 60 h of calculation time for each 6 s of simulation.

5. Postprocessing of experimental and simulation data

5.1. Calculation of bed height from experimental PIV data

To obtain the particle bed height at different positions in the chute from the PTV data, a postprocessing step is necessary. We assume that the bed surface can be defined as the height at which the visible track density (in the height direction) is highest. This is better than using an envelope around the top-most tracks, because such a measure would be dominated by positions of stray particles. To determine the local track density, the chute volume was divided into computational cells. For each cell, the number of tracks that pass through it was calculated, where tracks with a length of more than 10 positions were used. Subsequently, for each column of cells the local maximum of the track density was ascertained, and thus the height of the bed surface is known. We divided the entire chute in 250 (length) × 16 (width) × 200 (height)=800,000 cells. This division was chosen to strike the best balance between a small cell size in the height direction and a sufficiently large number of tracks per cell. The resulting error on the estimated bed height is typically the same as the error on the determination of the vertical (depth) position of the particles, i.e. 1.0 mm.

Fig. 4 (blue line) shows an example of the bed height obtained in this manner for a non-rotating chute, as a function of streamwise position and averaged over the width of the chute. Note that the measurements are still somewhat noisy. Therefore, a Gaussian filter of width 0.07 m was used to smooth the curve as shown in Fig. 4 (blue circles). This filter width is large enough to significantly smooth the data, yet small enough to still resolve the dependence of bed height on streamwise position.

5.2. Calculation of bed height in simulations

In the DEM simulations, the bed height was calculated via the center of mass (COM) height. In an ideal situation of homogenous packing, the bed height is exactly twice the center of mass height. However, we note that the bed height obtained from PTV measurements is basically a result of tracking the centers of particles. To enable a fair comparison between experiments and simulations, we should therefore base the bed height on the average height of the centers of the surface particles. As explained in Fig. 5, the bed height (h) can be obtained by subtracting half a particle diameter from twice the COM height.

5.3. Calculation of experimental surface velocity using PIV

We used particle image velocimetry (PIV) to determine the average two-dimensional velocity field of the visible surface particles. Pairs of images were postprocessed using the commercial software package Davis 8.0.3. A multipass algorithm was used with an initial interrogation area size of 32 × 32 pixels and a final non-overlapping interrogation area size of 16 × 16 pixels, which yielded an approximate displacement error of O(0.1) pixels. Due to the very high seeding density (particles per interrogation area) there were practically no outliers. Any remaining outliers were removed with a standard median filter. For rotating chute flow, there may be a significant sideways motion and parts of the chute may fall “dry”, meaning that only stray particles are present in

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**Table 2**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of chute</td>
<td>0.9 m</td>
</tr>
<tr>
<td>Width of chute</td>
<td>0.08 m</td>
</tr>
<tr>
<td>Height of chute</td>
<td>0.09 m</td>
</tr>
<tr>
<td>Inclination of chute</td>
<td>30°</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>3.2 kg/s</td>
</tr>
<tr>
<td>Particle type</td>
<td>Spherical glass</td>
</tr>
<tr>
<td>Particle diameter</td>
<td>3 mm</td>
</tr>
<tr>
<td>Particle density</td>
<td>2550 kg/m³</td>
</tr>
<tr>
<td>Coefficient of normal restitution</td>
<td>( e_{\text{sw}} = e_{\text{sw,0}} = 0.96 )</td>
</tr>
<tr>
<td>Coefficient of tangential restitution</td>
<td>( e_{\text{sw}} = e_{\text{sw,0}} = 0.33 )</td>
</tr>
<tr>
<td>Coefficient of friction</td>
<td>( \mu_{\text{Y}} = 0.10, \mu_{\text{H}} = 0.22 )</td>
</tr>
<tr>
<td>Total simulation time</td>
<td>6.0 s</td>
</tr>
<tr>
<td>Time step</td>
<td>( 2.5 \times 10^{-6} ) s</td>
</tr>
</tbody>
</table>

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**Fig. 3.** Snapshot of particle introduction at the inlet of the chute.

**Fig. 4.** Experimental particle bed height defined by the highest track density from PTV, for a non-rotating and rotating chute. The blue line represents the raw data from the post-processing, the blue circles, the smoothed bed height profile. Note that the smoothing was performed on all data, including positions near the inlet for \( z < 0 \). The much larger bed height near the inlet explains the apparent increase of the smoothed data in the range between 0 and 0.03 m. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
some regions. Masking was performed on these dry regions, meaning that no cross correlation analysis was applied in the masked region of the chute. After post-processing, a time-averaged velocity field was obtained from a sequence of instantaneous velocity fields obtained over a time interval during which the flow was steady. We found that the error in this velocity field was 0.2%, which is relatively small.

5.4. Calculation of experimental surface velocity using PTV

Similar to PIV, it is possible to determine the surface velocity profile using the PTV technique. From the known positions of tracer particle tracks at different time instances and the time difference in between two frames, we derived the velocity profile. Although PTV is a 3D technique, in these dense granular flows it is limited to tracking surface particles (those visible from the top). Thus the results (for the streamwise and spanwise components) are expected to match the surface particle velocity obtained from the PIV technique.

The procedure for obtaining Eulerian velocity profiles from the Lagrangian PTV tracks was as follows. (1) The tracks were smoothed using a (Matlab) loess filter to avoid irregularities. (2) The domain was divided into 250 (length) × 9 (width) columns. Coordinates of tracks inside each column were identified. (3) The distance between each consecutive point of a track in a column was found, and multiplied by the frame rate to obtain the velocity of that track section. (4) The average velocity for each column was calculated to obtain the surface velocity profile as a function of spanwise and streamwise position.

Each individual velocity measurement had a relatively high error of 1.0 m/s due to the uncertainty of 0.5 mm in particle position combined with the frame rate of 2000 Hz. However, the error in the estimate of the average velocity was much smaller. Treating each velocity measurement as an independent measurement, and using approximately 400–500 individual velocity measurements per column, we arrived at an error in the estimated average velocity of 0.05 m/s.

Note that the 3D-PTV technique also delivers information about the depthwise velocity component of the surface particles, which cannot be obtained from the current PIV experiments. Although another PIV camera could be installed viewing the chute perpendicularly from the side, this would still make it very difficult, if not impossible, to obtain the dependence of the depthwise velocity of the surface particles on their spanwise position inside the chute. As we will show, this is very relevant for rotating chute flows. Another option would be to use stereo-PIV (SPIV), which is routinely used in fluid dynamics experiments using a laser sheet to highlight a particular section in a fluid flow (Wieneke, 2005; Dreyer et al., 2014). However, this is not easily extensible to granular flows, leaving PTV as the best option to obtain the desired data.

5.5. Calculation of surface velocity in simulations

To enable a direct comparison between experimental and simulated surface velocity, a particle velocity field is calculated also in our simulations. It is important to only include those particles that would be optically visible from above the chute, because in the experiments the camera was mounted perpendicularly above the chute at a large distance. Specifically, in our simulations, the surface velocity is calculated by time-averaging the velocity of the topmost 6 particles in each streamwise and spanwise column of 5 mm × 6 mm. The choice for this number of particles was made because at the given particle diameter of 3 mm, it is expected that at most 6 particles will be visible from above the chute in each of the computational columns. We have checked the dependence of the surface velocity measurements on the number of topmost particles used, and found only a negligible influence for particle numbers ranging from 3 to 12.

6. Experimental results and discussion

6.1. Repeatability of PTV experiments

To investigate the repeatability of the PTV measurements, as well as to rule out possible effects of wear and tear of the glass particles, some experiments have been performed twice: once at the start of our experimental series, and once near the end. Fig. 6(a) shows the average streamwise surface particle velocity along the centerline of the chute as a function of the streamwise position, for a non-rotating and rotating chute. Fig. 6(b) shows the average bed height along the centerline of the chute as a function of the streamwise position, for a non-rotating chute.
explained in Section 5.1, the bed height calculated in the PTV ultrasonic sensor (red symbols) for a non-rotating chute. These chute using PTV measurements (blue symbols) and the electronic measurements (green). Close to the inlet, acoustic noise from the particles dropping onto the particle bed is confusing the ultrasonic signals, explaining the initial deviation. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

The results show that the repeatability of our PTV experiments is satisfactory.

6.2. Comparison of bed height using PTV and the ultrasonic sensor

Fig. 7 shows the bed height as measured in a non-rotating chute using PTV measurements (blue symbols) and the electronic ultrasonic sensor (red symbols) for a non-rotating chute. These experiments were performed with an interval of 4 months. As explained in Section 5.1, the bed height calculated in the PTV experiments is based on the maximum density of visible particles in the height direction, whereas the ultrasonic sensor measures the bed height over a circular area of 28 mm in diameter. To enable an honest comparison, we have averaged the PTV data over the same area. We note that in the initial section, up to 0.2 m from the chute entrance, the electronic height sensor measurements were somewhat affected by the noise (sound) generated by particles dropping from the hopper onto the chute surface. This may explain the deviations in the initial section. In the remaining section of the chute, the electronic height sensor measurements follow the PTV measurements quite closely, with an offset of the order of one particle radius (1.5 mm). This bias can be explained by the fact that sound waves scatter off the top of the particles, while PTV is tracking the centers of particles, as explained in Fig. 5. Overall, the agreement between the two independent experimental techniques is very satisfactory.

6.3. Comparison of streamwise velocity using PTV and PIV

Fig. 8 shows the averaged streamwise surface particle velocity along the center-line of the chute using PTV measurements (blue) and PIV measurements (green), for a non-rotating chute (lines) and a chute rotating at 24 rpm (circles). As observed in our previous work (Shirsath, 2013), the granular flow decelerates roughly in the first half of the chute and accelerates in the second half (both relative to the non-rotating case), which we attributed to a changing balance between Coriolis and centrifugal forces. For the purpose of this paper, it is encouraging to see that the two independent measurement techniques, one based on Lagrangian tracking of tracer particles and the other based on spatiotemporal correlations of all particles, are in good mutual agreement.

In the next section we will show more detailed comparisons between PTV and PIV measurements, but also include our DEM simulation results to validate the simulation method.

7. Validation of DEM using experimental measurements

Having performed a cross-validation of independent experimental measurements, we now use these measurements to validate our discrete element model. Fig. 9 shows from a top view: the experiment, the PIV velocity field, a collection of PTV particle tracks, and the simulation, respectively, for monodisperse glass particles in a non-rotating chute (left) and a chute rotating at 24 rpm (right). All results are in qualitative agreement with each other. The major effect of chute rotation is a sideways deflection of the particle stream due to Coriolis forces present in the frame of reference co-rotating with the chute. In the next sections we will present a detailed quantitative comparison.

7.1. Bed surface height

Fig. 10 shows the full bed surface as a collection of PTV particle tracks (left) and the full bed surface obtained from our DEM simulations (right) for non-rotating (top) and rotating (bottom) chutes.

Fig. 11 provides a more detailed comparison, showing the bed surface height as a function of position along the length of the chute, for a chute rotating at various rotation rates. The bed height is averaged over 5 mm wide sections of the chute, for three different width-wise positions in the chute (on the left, center and right-hand side). The simulation results (lines) are compared with the PTV experimental results (symbols). For the non-rotating chute, the bed height continuously decreases along the length of the chute, and the results are nearly indistinguishable for the different width-wise positions. As the rotation rate of the chute increases, the bed height increases on the right side of the chute and decreases on the left side. Moreover, at higher rotation rates we observe a maximum in the height as a function of streamwise position at the right side of the chute. Deviations between simulation and experiments are observed at the right side of the chute (blue lines) at higher rotation rates. This is consistent with our previous observation that the precise manner of particle introduction is important for the first section (approximately 0.2 m) of the chute. Given the experimental error of approximately half a particle diameter, these simulation results are in good agreement with the experimental measurements.
7.2. Streamwise surface particle velocity

Fig. 12 shows the streamwise surface particle velocity as a function of width-wise position at four different streamwise positions, for rotation rates 0 rpm (non-rotating chute), 8 rpm, 16 rpm and 24 rpm. The streamwise positions are at $z=0.0$ m, $0.2$ m, $0.4$ m and $0.6$ m. The simulation results (lines) are compared with the experimental results of PIV (filled symbols) and PTV (open symbols). Generally we find good agreement between the simulations and both sets of experimental results. Some deviations between the PIV and PTV measurements are visible at higher rotation rates. This is caused by the sideways motion of the...
granular particles, leaving one side of the chute empty with occasional stray particles. These stray particle regions are masked out from our PIV analysis, while the stray particles are still processed up by our PTV analysis. The observed deviations therefore correspond to velocities of very low number of particles.

7.3. Spanwise surface particle velocity

Fig. 13 shows the spanwise surface particle velocity along the width of the chute at different cross sections in the length of the chute. The simulation results are compared with the experimental results for different rotation rates of the chute. We observe that the magnitude of the spanwise velocity first increases and then decreases for consecutive streamwise positions. This corresponds to the process of sideways motion induced by Coriolis forces, which is finally stopped by the compaction of the granular flow against the side wall. The maximum spanwise velocity increases with increasing rotation rate. Some deviations between simulation and experiments are observed at the start of the chute (for \( z \leq 0.2 \) m) because the way of introducing particles in the simulations is different from experiments. In all other cases, a rather good agreement is found between experiments and simulations. Note that previously we estimated the error in the average velocity obtained from PTV to be of the order of 0.05 m/s. This is consistent with the fluctuations observed in the PTV data, especially for the non-rotating case where no sideways velocity is expected.

7.4. Depthwise surface particle velocity

Fig. 14 shows the depthwise surface particle velocity along the length of the chute for different spanwise sections of the chute, as obtained from the PTV experiments. Note that for the non-rotating
chute, the depthwise velocity is high near the inlet of the chute and gradually decreases to zero. For rotating chutes, there is a marked difference between relatively small depthwise velocities measured at the left side of the chute (blue lines) and relatively large depthwise velocities at the start of the right side of the chute (red lines). This is consistent with the observed bed height profiles from our simulations and PTV tracks. We confirmed that the slope of the bed height is consistent with the ratio of streamwise to depthwise velocity (not shown). This confirms that the particles move parallel to the bed surface.

8. Conclusions

In this paper, a three-dimensional particle tracking technique was applied to study bed height and surface particle velocities of granular flows in rotating chutes. The 3D-PTV experiments show very good repeatability. A cross-validation of the particle bed height and the two-dimensional velocity field was performed by comparing independent results obtained using an electronic ultrasonic height sensor and particle image velocimetry. All experimental results are in good mutual agreement.

The 3D-PTV technique has several advantages over other existing techniques. The advantages relative to the use of an electronic height sensor for the measurement of the bed height are, first, a better spatial resolution of the bed height profile and, second, a great reduction of the number of necessary experiments (unless a large number of height sensors is used). The advantage of 3D-PTV relative to the use of particle image velocimetry (PIV) is the measurement of the third (depthwise) component of the surface particle velocity. An overall advantage of 3D-PTV is that it can provide the measurements of particle bed height and surface particle velocity field within one and the same experiment.

We have used the experimental results to further validate our discrete element model. We have shown that the simulation
model is capable of predicting the evolution of the granular flow both qualitatively and quantitatively.

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