Frequency Limitations of First-Order \( g_m \)-RC All-pass Delay Circuits

Seyed Kasra Garakoui, Eric A. M. Klumperink, Bram Nauta, Frank E. van Vliet

Abstract: All-pass filter circuits can implement a time delay, but in practice show delay and gain variations versus frequency, limiting their useful frequency range. This paper derives analytical equations to estimate this frequency range, given a certain maximum allowable budget for variation in delay and gain. We analyze and compare two well-known \( g_m \)-RC 1st-order all-pass circuits, which can be realized compactly in CMOS technology and relate their delay variation to the main pole frequency. Modeling parasitic poles and putting a constraint on gain variation, equations for the maximum achievable pole frequency and delay variation versus frequency are derived. These equations are compared to simulation and used to design and compare delay cells satisfying given design goals.

Index Terms: All-pass filter, delay, true time delay, phase shift, phase shifter, filter optimization, bandwidth, frequency range.

I. INTRODUCTION

Analog time delay circuits have several applications, for instance compensating delay differences between signal paths, broadband beam forming [1] and equalizing the communication channel for wireline communication [2]. Ideally such delay circuits should have both a constant unity gain and a well-defined constant delay, which does not vary with frequency. However, practical delay circuits do show frequency dependent gain and delay variations. This frequency dependence affects the functionality of systems which exploit delay circuits, limiting their accuracy. For example in time-delay based phased-array antenna systems, a frequency dependent time delay causes a frequency dependency in the beam direction (“beam squint”) [3,4].

CMOS is often the desired technology for the implementation of mixed-signal systems. At radio frequencies, OPAMPs are impractical, and time delays are typically implemented either transmission lines [5], LC delay lines [1] or all-pass \( g_m \)-RC delay circuits [2,6,7]. In this paper we focus on circuits which can be implemented in standard CMOS IC-technology at low area cost and low supply voltage. Transmission lines in CMOS require very long (lossy) metal lines to produce a significant amount of delay, while LC delay lines need on-chip inductors. The \( g_m \)-RC all-pass delay circuits proposed in [2,6,7] can produce a given amount of delay much more compactly than inductor based delay cells. Although many trade-offs exist, for instance in achievable frequency, noise, linearity and power consumption, \( g_m \)-RC all-pass circuits are clearly area and hence cost effective and will be the focus of this paper.

An ideal 1st order all-pass filter has a pole and zero, and can be written as:

\[
H(s) = \frac{1 - \frac{s}{2\pi f_p}}{1 + \frac{s}{2\pi f_p}} \tag{1}
\]

where \( f_p \) refers to the pole-frequency. Note that the pole and zeros are positioned at \( \pm f_p \), resulting in twice the phase and delay of a single-pole system. Also, the gain is 1 and frequency-independent. Figure 1 shows the phase and gain of eqn. (1), in comparison to an ideal time-delay cell. The time delay at an operating frequency \( f_0 \) is equal to \( \tau = \phi(f_0)/(2\pi f_0) \). As figure 1 shows, the delay of a first order all-pass cell is frequency dependent and varies with \( f_0 \).

In general, delay variations [8], but also gain variations limit the useful frequency range. What is acceptable depends on system requirements (see for instance [3,4]), and we will assume a maximum allowed gain and delay variation budget. This paper provides a method to analyze the achievable frequency range of delay circuits given such a budget.

In literature we found several delay circuits, but no comparison of their relative merits, nor a design method to maximize the useful frequency range. This paper aims at filling this gap.

As low level circuit details critically affect delay cell performance, we will analyze and compare two well-known voltage-mode all-pass circuits. One is the “classical” all-pass delay circuit described in [6], but with much older roots at least dating back to [9]. We will compare this to the “Buckwalter” cell structure proposed in [2]. The circuit of [7] is not considered further, as it is a current-mode circuit complicating comparison, and as it uses 3 stacked transistors which is less suitable for low supply voltages.

We will propose an analysis method which also holds for the practical case where a delay cell operates in a cascade of similar delay cells. The input impedance of the next cell will then load the previous one, while an extra capacitive load (\( C_L \)) may also be present. The analysis partly builds on [8], where the criterion \( f_{\sigma=0} \) is introduced to quantify variations of delay
versus frequency. This figure of merit has some properties [8] that we will exploit, which are briefly summarized below.

As shown in figure 2, $f_{\varphi=0}$ is the frequency where the tangent to the phase transfer function at $f_0$ crosses the frequency axis ($\varphi=0$). For an ideal time delay cell, $f_{\varphi=0}/f_0=0$ and for an ideal phase shifter $f_{\varphi=0}/f_0=-\infty$ [8]. For a practical delay cell with non-linear phase transfer function a low value of $f_{\varphi=0}/f_0$ is desirable. In general, $f_{\varphi=0}/f_0$ values can be found from the phase transfer function as [8]:

$$f_{\varphi=0,\text{cell}}/f_0 = 1 - \frac{\varphi_{\text{cell}}(f_0)}{f_0} \left( 1 + \frac{f_0}{f_p} \right)$$

Now, the relative delay variation $\Delta t_d/t_0(f_0)$ for a frequency variation $\Delta f$ around $f_0$ is given by [8]:

$$\Delta t_d(f_0) = \frac{f_{\varphi=0,\text{cell}}}{f_0} \frac{\Delta f}{1 - f_{\varphi=0,\text{cell}}/f_0}$$

Clearly, if $f_{\varphi=0}/f_0<<1$, then $\Delta t_d(t_0(f_0))=0$ which means the circuit approximates an ideal delay over frequency band $\Delta f$.

If we apply (2) to the phase transfer function of the ideal 1st order all-pass cell (1), we find:

$$f_{\varphi=0,\text{cell}}/f_0 = 1 - \arctan \left( \frac{f_0}{f_p} \right) \left( 1 + \left( \frac{f_0}{f_p} \right)^2 \right)$$

Figure 3 plots numerical values for (4) versus the operating frequency $f_0$, normalized to pole frequency $f_p$. Also, $f_p \tau$ is shown, which varies slightly around 0.25 for $f_0=f_p$ (phase shift -\pi/2). For low relative delay variation at given $\Delta f$ and $f_{\varphi=0}$ (3) asks for low $f_{\varphi=0}/f_0$, i.e. large $f_p$. However, as $f_p \tau$ is around 0.25, less delay per cell results (roughly $\propto 1/f_p$). Fortunately, cascading cells allows for more delay at constant relative delay variation. If cells are identical, $f_{\varphi=0}$ of the cascade is equal to that of a cell [8]. Hence analyzing $f_p$ of a single (loaded) cell is sufficient to characterize a cascade of delay cells with respect to delay variation.

In this paper we will show how the maximum value of $f_p$ is limited by circuit topology and technology parameters, where also gain variations induced by parasitic poles will be considered. Note that previous work [8] only modeled ideal low-pass RC and LC delay cell behavior, no all-pass cells. Also, no gain variation effects nor parasitic loading effects were modeled, which is way too optimistic.

The maximum achievable $f_p$ will now be analyzed for the “classical” delay circuit in sections II and the “Buchwalter” delay circuit in section III. Section IV will verify analysis by simulation, and compare the relative merits of the circuits. Section V will present the conclusions.

II. ANALYSIS: THE CLASSICAL DELAY CIRCUIT

Figure 4 shows the classical all-pass delay cell [6][9]. Instead of resistors, diode connected MOSFETs or triode MOSFETs may also be used, but resistors are preferred for linearity reasons, less parasitic capacitance than MOSFETs and because they required low voltage headroom.

Eqn. (5) shows an approximate transfer function.

$$\frac{V_{\text{out}}}{V_{\text{in}}} \approx -\frac{g_m}{g_m + G_A} \cdot \left( 1 - \frac{R_H}{g_m + G_A} \right) \left( C_H + C_{\text{in}} + C_L \right)$$

$C_m$ is the input capacitance of the next stage. If each delay cell is cascaded with identical delay cells, then $C_m=2C_{gd}$ (due to the Miller effect on $C_{gd}$). Eqn. 5 differs from the transfer function (1) because its DC-gain is less than one and the absolute value of the pole and zero differ from each other, causing attenuation at high frequencies (see Figure 5). Still, eqn.5 can approximately imitate the transfer function (1) so we can re-use (4) provided that two conditions are satisfied:

1) $\frac{g_m}{g_m+G_A} \approx 1$, and
2) $\left( R_H + \frac{2}{g_m+G_A} \right) (C_H + C_{\text{in}} + C_L) \approx R_H C_H$.

The second condition ensures proximity of the absolute value of the pole and the zero frequency, which keeps the amount of frequency dependent gain attenuation small.
We will now assume the design requirements are:
1) DC gain $A_v0$, ($0<A_v0<1$)
2) high-frequency attenuation $\leq \Delta H_p$

By substituting these values in (5), we get:

$$R_a \geq \frac{A_v0}{1-A_v0 g_m} \quad (6)$$

$$R_H \geq \frac{2 A_v0}{g_m \left( \sqrt{2(1-\Delta H_p)^2} - 1 \right)} \quad (7)$$

$$C_{Hc} \geq \left( \frac{1}{\sqrt{2(1-\Delta H_p)^2} - 1} \right) \quad (8)$$

Based on (7) and (8), the condition on the pole and zero frequencies for the “Classical” delay circuit becomes:

$$|f_{p,c}| \leq \frac{1}{2 \pi R_H C_{Hc}} - \frac{1}{2 \pi R_H C_{Hc}} \quad (9a)$$

$$|f_{z,c}| \leq \frac{1}{2 \pi R_H C_{Hc}} \quad (9b)$$

Not that (9a) gives the maximum possible pole frequency, i.e. the lowest delay variation (see (4) and figure 3).

III. ANALYSIS: THE BUCKWALTER DELAY CIRCUIT

Figure 6 shows the all-pass circuit proposed by Buckwalter [2] implemented using MOSFETs and resistors.

![Figure 6. Buckwalter circuit](image)

As we mentioned before the circuit is cascaded with identical circuits therefore the circuit is loaded with impedance equal to its input impedance and an extra load capacitance $C_{L}$. The input impedance can be simply approximated evaluating the Miller effects on capacitances $C_{Hc}$ and $C_{gs1}$: $Z_m = 2C_B + C_{gs1}/(1 + g_m R_S) = 2C_B$. During the rest of the calculations the value of $C_{gs1}$ is absorbed inside $C_B$. $C_{prs} = C_{db1} + C_{gs2}$. The approximate transfer function of the circuit is eqn.10.

$$V_{out} \approx \frac{\frac{1}{g_m} + R_S}{R_D} \left( 1 - \frac{1}{g_m R_S} \right) \quad (10)$$

$$\left( 1 + \frac{1}{R_D g_m2} \right) \left( 1 + \frac{C_{prs} + C_L}{C_B} \frac{R_D g_m2}{1 + \frac{2}{2 R_D g_m2}} \right)$$

The right side of the (10) contains 2 parts. The first part is the DC-gain and the second part is approximately the all-pass transfer function. The initial phase shift at DC is $\pi$, which we do not consider it in our calculations because in processing differential signals, the $\pi$ phase shift can be compensated with interchanging the output signals [2]. To design the delay circuit we again aim at approximating transfer function (1). For the unity DC-gain the following condition must be satisfied: $R_D = R_S + 1/g_m1$. Figure 7 shows the gain transfer function of the Buckwalter delay circuit with unity DC-gain.

![Figure 7. Voltage gain of the Buckwalter delay cell](image)
Again, (13a) provides a estimate of the maximum possible pole frequency of the Buckwalter delay circuit, and (3) and (4) and figure 3 relate this to delay variation.

IV. VERIFICATION AND DESIGN EXAMPLES

To verify analysis and exemplify how the analytical equations can be used during design, we will address two design questions: A) which of the delay cells achieves the highest \( f_p \) for a given power budget? B) What is the power consumption for each circuit provided that they fulfill the same delay and noise figure requirements?

We will use a CMOS process UMC180nm supply voltage equal to 1.8 Volt.

A. Maximum \( f_p \) for a given power budget

Suppose the design requirements for a delay cell in a cascaded chain are: DC-gain\( >1 \text{dB} \), \( \Delta H_p < 1 \text{dB} \), and we allow a maximum DC current of 3.5mA. Assume also two loading cases: \( C_L \) is 0 or 2pF. We aim at finding the best circuit w.r.t. delay variations and hence compare the maximum achievable \( f_p \). To reduce the channel length modulation in transistors, their lengths are chosen 240nm. The overdrive voltage of all transistors is chosen equal for both circuits (\( V_{GS,OV} = 75 \text{mV} \)), so that \( f_T \) of the transistors is equal. For Buckwalter’s cell, the size of \( M_2 \) is chosen 15 times of \( M_1 \) to satisfy (11), even for \( R_c = 0 \) (table I).

<table>
<thead>
<tr>
<th>Circuit</th>
<th>W/L [( \mu \text{m} / \mu \text{m} )]</th>
<th>( V_{GS,OV} ) [mV]</th>
<th>( f_T ) [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Classical”</td>
<td>420.76/0.24</td>
<td>75</td>
<td>12.4</td>
</tr>
<tr>
<td>Buckwalter</td>
<td>(W/L)(_1) = 26.29/0.24, (W/L)(_2) = 394.46/0.24</td>
<td>75</td>
<td>12.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_L ) [pF]</th>
<th>Classic 0 fF</th>
<th>Classic 2pF</th>
<th>Buckw. 0 fF</th>
<th>Buckw. 2pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{NC} ) [mA]</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>#NMOS</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( f_T,\text{NMOS} ) [GHz]</td>
<td>12.4</td>
<td>12.4</td>
<td>12.4</td>
<td>12.4</td>
</tr>
<tr>
<td>( \Delta H_p )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \Delta H_p ) [dB]</td>
<td>1.29</td>
<td>0.84</td>
<td>1.25</td>
<td>0.97</td>
</tr>
<tr>
<td>( A_{\text{DC}} ) [dB]</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_{\text{DC}} ) [dB]</td>
<td>-1.47</td>
<td>-1.47</td>
<td>-0.28</td>
<td>-0.28</td>
</tr>
<tr>
<td>( f_p ) calc. [MHz]</td>
<td>199</td>
<td>29.3</td>
<td>520</td>
<td>277</td>
</tr>
<tr>
<td>( f_p ) sim. [MHz]</td>
<td>201</td>
<td>32.2</td>
<td>546</td>
<td>288</td>
</tr>
<tr>
<td>( V_{\text{out}} )</td>
<td>2.7</td>
<td>2.7</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>( V_{\text{out}} )</td>
<td>16.5</td>
<td>17.5</td>
<td>14.8</td>
<td>17</td>
</tr>
</tbody>
</table>

For these transistor sizes, and \( C_{ad} \) for “Classical” is 173fF and \( C_{ad} \) for “Buckwalter” is around 152fF. Based on (8) and (12) and \( C_L = 0 \) we find: \( C_{L} \geq 2434 \text{ fF} \) and \( C_{L} \geq 943 \text{ fF} \), while for \( C_L = 2\text{pF} \), we find \( C_{L} \geq 16.508 \text{ pF} \) and \( C_L \geq 1.771 \text{ pF} \) (in the Buckwalter circuit the effect of \( C_L \) is reduced by the buffer stage). The values for resistors calculated based on (7) are \( R_S = R_S = 170 \Omega \) and \( R_P = 252 \Omega \), while for the Buckwalter circuit we find: \( R_S = 0 \), \( R_P = 249 \Omega \) (to have unity gain condition at DC: \( R_P = R_S + 1/g_{m1} \)). Table II shows calculated and simulated values for each delay cell for the two loading conditions, which match good enough for first-cut circuit design. As noise and linearity often are also important, we also added simulation results for noise and IIP3 (with 50\( \Omega \) as reference) for an operating frequency of \( f_p \) where \( \Delta H_p \) has been estimated and verified. Overall, the Buckwalter cell achieves better higher \( f_p \) and is less sensitive to \( C_L \).

Figure 8 shows the simulated phase and gain plots of the delay cells. The values of \( A_{V_0} \) (the DC gain) and \( \Delta H_p \) (gain drop at the pole frequency) are shown in the figures, where the pole frequency is defined as the frequency where the phase has dropped 90° with respect to DC. Note that, although circuit parasitics affect the transfer function, the phase characteristic roughly resembles that of a 1st order all-pass filter. As predicted, the “Classical” circuit has attenuation at low frequency, but the Buckwalter circuit can have quite near to unity DC-gain. Clearly, the Buckwalter circuits achieve the best frequency range, with also less overall attenuation.
If we compare these results to that of a single-pole low-pass circuit, we achieve double the delay using an all-pass cell, and about 1dB gain-drop instead of 3dB at the pole frequency.

<table>
<thead>
<tr>
<th>TABLE III: PARAMETERS FOR EQUAL DELAY AND 10DB NOISE FIGURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/L</td>
</tr>
<tr>
<td>Classical</td>
</tr>
<tr>
<td>Buckwalter</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

B. Power comparison for equal delay cell requirements

Suppose now the following properties are desired: an operating frequency f_0=100MHz, a relative delay variation of 5% for Δf=±10MHz around f_0, a DC-gain >-1dB, ΔH_p<1dB, and NF=10dB referred to 50 Ohm, while we want to calculate the achievable delay per cell.

To obtain the maximum delay per cell, we choose the minimum f_p that just satisfies eqn.(3) [8] for the requirements mentioned above. For design, suppose the initial sizes of the transistors are as in Table II, then from f_p we find the value of capacitors. Using admittance scaling [10] to satisfy the noise figure requirements, we multiply all W and capacitor values by α and divide all resistor values by α. With this method the circuit delays don’t change, however the noise figure will change proportional to the power consumption (constant SNR/Power [10]). The power consumption also strongly depends on the overdrive voltage of the transistors. Substituting the delay variation of 5% and relative bandwith of ±10MHz into eqn.3 results in f_p/f_0=1.4. Therefore the minimum value of f_p of the delay cells must be 100/1.4=71.4MHz. This requires equal f_p for both circuits, i.e. R_pC_p=1/2f_p=2.23e-9 and also: R_pC_p=1/2f_p=2.23e-9. To bring the noise figure to 10dB for both, the values of resistors and capacitors will be: R_p=173.4Ω, R_p=2572Ω, C_p=8.67pF and R_0=321.2Ω, C_p=6.94pF. Table III shows the resulting aspect ratio and DC current consumption of both circuits. The achievable delay with both circuits now is 3.025 ns/cell at f_0=100MHz. For the same noise figure the Buckwalter cell consumes 20% less DC current compared to the Classic circuit. Also the majority of its current is mainly consumed by the buffer transistor. For some applications the loading capacitance may be small and it might be possible to remove the buffer part of the Buckwalter circuit to reduce its power consumption.

V. SUMMARY AND CONCLUSION

A method for analyzing the maximum useful frequency range of 1st order all-pass delay cells was introduced. It has been used to analyze and compare two well-known g_mRC delay cells, the “Classical” and “Buckwalter” all-pass cell. The analysis holds for single and cascaded identical cells. To this end, the cells have been analyzed is a self-loaded content with also an arbitrary extra loading capacitance. The resulting transfer function deviates from ideal delay-cell behavior in two key aspects: both the gain and delay are frequency dependent. Design boundary conditions were derived for each all-pass circuit to keep the gain variation below a specified maximum and ensure that the shape of the transfer function still resembles the simple 1st order all-pass function eqn. (1), which is characterized by one pole frequency f_p. The design boundary conditions were then expressed as constraints on the maximum achievable pole frequency f_p. Substituting f_p in eqn. (4), eqn. (3) can now be used to estimate the amount of delay variation as a function of the frequency variation Δf around the nominal operating frequency f_0.

For any single 1st order all-pass delay cell, or approximation thereof, a larger value of f_p renders smaller delay variations over a given frequency range Δf around f_0, but also smaller delay (see Figure 3). When cascading multiple cells, each cell needs to realize less delay, and hence can have higher f_p, resulting in less relative delay variations (3). This thus results in a larger useful frequency range (but more chip area and power consumption). The equations in this paper model the trade-offs between delay-cell pole frequency f_p, center frequency f_0, frequency range Δf and delay variation, while keeping amplitude variations within a budget and allowing for improving performance by cascading cells.

To exemplify the usefulness of the analysis, it was applied to two delay cells to compare their relative performance and estimate their delay variation versus frequency. With the help of the derived design equations, an optimization of f_p was illustrated, keeping delay and power constraints fixed, comparing results with and without extra capacitance. Secondly, the analysis results were exploited to achieve equal delay with different circuits, under the condition of fixed noise performance, while comparing power dissipations. Similar analysis can be done for other kinds of 1st order gm-RC all-pass delay cells.

REFERENCES
