Lowering the SNR-Wall for Energy Detection Using Crosscorrelation

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Abstract—Spectrum sensing is a key enabler of cognitive radio, but generally suffers from what is called an SNR-wall: a minimum SNR below which it is impossible to reliably detect a signal. For energy detection, which has the advantage that it requires no knowledge of the signal, the SNR-wall is caused by uncertainty in the noise level. Crosscorrelation has been suggested as a possible means to obtain higher sensitivity, but has received little attention in the context of noise uncertainty. The idea of crosscorrelation is to have two receive paths, where each path independently processes the signal before they are combined, such that the noise added to the input signal at the individual paths is largely uncorrelated. In this paper we mathematically quantify the SNR-wall for crosscorrelation, showing that it scales linearly with the amount of noise correlation. This lower noise correlation results in a higher sensitivity, significantly better than for autocorrelation. Equations are derived that can be used to estimate the benefit over autocorrelation and to estimate the measurement time for a required probability of detection and false alarm.

Index Terms—cognitive radio, crosscorrelation, dynamic spectrum access, energy detection, noise uncertainty, radiometer, SNR-wall, spectrum sensing

I. INTRODUCTION

COGNITIVE RADIO (CR) is a promising concept to improve the efficiency of spectrum use. CR requires a radio to sense the spectrum to find unoccupied frequency bands, which can then be used for data transmission. Many different types of spectrum sensing exist, in which there usually is a tradeoff between achievable sensitivity, required knowledge of the signal to be detected, and computational complexity.

A key limitation of spectrum sensing is that it suffers from a so-called SNR-wall: a minimum SNR below which a signal cannot be reliably detected, i.e., the probability of false alarm and/or the probability of missed detection becomes larger than 1/2 [1]. This SNR-wall is caused by properties crucial to the sensing technique: for energy detection it is the uncertainty in the noise level, whereas for pilot detection it is the limited coherence time [1]. In [2] a modification to the primary signal is proposed to eliminate the SNR-wall. While conceptually interesting, this is not very practical (at least on the short term), as existing primary users all need to be adapted. In contrast, we concentrate on a technique that is independent of additions to the primary signal.

For energy detection, the decision whether a signal is present or not is based on the difference between the measured power level and the (estimated) noise power level. The noise level is composed of noise from the physical channel and noise from the receiving device. This noise level can be estimated, but with limited accuracy, e.g. due to the fact that the antenna noise varies as a function of weather, the noise of the receiver may vary over frequency and during operation and the noise level estimation itself will always have some error [1], [3]. This results in some uncertainty range of the noise level. When the SNR is too low, the measured total power (only noise, or noise + signal) will be within this uncertainty range, resulting in either many false alarms or many missed detections. In other words, reliable detection becomes impossible.

From existing literature, the SNR-wall for practical noise uncertainty is $-6$ dB, which is still far off from the detection limit of SNR=$-15$ dB as required by IEEE 802.22, a standard for CR WRAN in the TV-bands. Other sensing techniques exist to achieve detection under such low SNR-conditions, but are in general more computationally intensive, and require knowledge of the signals to be detected. Examples are the exploitation of the strong carrier present in an ATSC signal [4], or the cyclostationarity present in many modulated signals [5]. As frequency bands already now contain signals with different modulation techniques, energy detection is preferred because no knowledge of the modulation is required.

A possible solution to mitigate the noise uncertainty problem for energy detection in low SNR-conditions is crosscorrelation, as briefly mentioned by [3], [6]. Crosscorrelation has a low computational complexity and does not require knowledge of the signals to be detected. It has been exploited in [7] to reduce the associated noise penalty of passive attenuation to obtain high linearity. In all cases an important question is how noise uncertainty affects detection performance. A mathematical derivation is given in [8], but it lacks the link to practical applications and easily usable engineering approximations. The contribution of this paper is threefold:

- It links the effect of noise uncertainty in crosscorrelation to the SNR-wall for primary user detection, backed up by simulations;
- It provides an explicit comparison to autocorrelation-
based energy detection, showing improvements in measurement time and higher sensitivity;

- It provides approximate expressions relating detection performance, bandwidth and measurement time, which can be useful for system engineers.

The next section briefly describes some alternative energy detection algorithms to overcome the noise uncertainty. Section III introduces the system model used for crosscorrelation, while section IV derives the SNR-wall for this system model. Section V shows the potential of crosscorrelation in a realistic example, and section VI discusses the results obtained, while conclusions are drawn in section VII.

II. RELATED WORK

The problem of energy detection of unknown signals buried in noise was addressed by [9], in which the noise power was assumed to be exactly known. In case of exact knowledge of the noise power, a longer measurement time reduces the variance, such that signals at any SNR can eventually be detected. In the presence of noise uncertainty, [3] showed that there is a limit to the signal power that can be detected, even for infinite measurement time. In the context of CR, this limit is called the ‘SNR-wall’, a term coined by Tandra & Sahai [10]. Many recent publications have sought to overcome this SNR-wall in one way or another.

In [11], oversampling is proposed to distinguish between noise and signal. Due to oversampling, the signal component of subsequent samples will be highly correlated, while the noise samples will not be. The same assumption is used in [12] to be able to distinguish between the autocorrelation functions (acfs) of the signal and the noise. However, the noise itself may not be perfectly white, which results in correlation in subsequent samples, or, equivalently, an acf that can be non-zero over large time-shifts, and hence obscure the signal to be detected in low SNR conditions. Moreover, both approaches require a more wideband frontend and faster analog-to-digital converters (ADCs) to accommodate oversampling, resulting in a higher power consumption even when the system is not performing spectrum sensing.

An alternative technique is based on eigenvalues calculated from a covariance matrix [13]. Although [13] uses a general model with multiple receivers, it is always explicitly assumed that the noise samples of the different receivers are independent. However, [14] shows that this is (approximately) true only under certain conditions. An extension to the work of [13] is provided in [15], but also there it is assumed that all noise samples are completely independent. The requirement on independence of the noise samples in the different receivers is undesirable, as these are difficult to meet in practice. We explicitly take noise correlation into account.

The limitations of crosscorrelation spectrum sensing have been explored in [8] based on a relatively extensive model. The conclusions drawn were only qualitative, because the final expressions obtained are not easily interpreted. We will start from a simpler model, which enables us to derive simple equations that give insight into the dependencies and enable system engineers to directly apply them.

Cooperative sensing is often mentioned to improve performance by combining data from several nodes using some algorithm to obtain a decision whether the band is free or not. This scheme is still possible with crosscorrelation; the results of individual nodes (obtained by using crosscorrelation, or some other spectrum sensing algorithm) can be combined to obtain a better quality decision. Fundamentally, the SNR-wall of individual detectors limits performance of cooperative sensing, so crosscorrelation can provide benefits here as well.

III. CROSSCORRELATION SYSTEM MODEL

The crosscorrelation system model is shown in fig. 1a and is based on the system model of [16]. It consists of a signal source $s$ and three noise sources $n_{\text{cor}}$, $n_1$ and $n_2$. It is assumed that the noise sources are independent complex zero-mean white Gaussian noise processes within the band of interest. The real and imaginary parts of each process have equal average power and are independent. The same assumptions are used for $s$, as this is a good approximation of the signals transmitted by OFDM systems, which is not only often proposed as the modulation technique to use in CR-technology, but is also in use by many wireless standards, including DVB, LTE, IEEE 802.11 (WiFi), IEEE 802.16 (WiMAX) and IEEE 802.22. We revisit some of these assumptions in section VI.

We use capital letters to denote the stochastic processes, e.g. $N_1$ is a stochastic process of which $n_1(t)$ is a realization.

The receiver chain, which typically includes an LNA, mixer, amplifiers, prefilters and an ADC, is modeled as a device that only adds some noise. It is assumed that the output of the receiver chain is complex, which allows the modeling of quadrature receivers. For a crosscorrelation system (part of) the receiver chain is duplicated, where uncorrelated noise is added in each of the two paths. The signal plus correlated noise $x$ is filtered in each path by a filter with bandwidth $W$ and then sampled at the Nyquist rate, resulting in the complex receiver outputs $r_1$ and $r_2$. The energy detection is performed by multiplying $r_1$ and $r_2^*$ ($r_2^*$ denotes the complex conjugate of $r_2$).
of \( r_2 \), and averaging the result. Note that for autocorrelation, \( r_1 r_2^* \) is equivalent to \( |r|^2 \), see fig. 1b. This final output \( y \) is used to decide whether a signal is present or not.

For the output of each path, for \( r_1 \) and \( r_2 \), we can define \( \text{SNR} = P_s / N_0 W \), where \( P_s \) is the power of \( s \) and \( N_0 \) is the PSD of the noise. This SNR includes contributions from \( v_{\text{corr}} \) and either \( n_1 \) or \( n_2 \). For simplicity, we assume that the PSDs of \( n_1 \) and \( n_2 \) are equal, which would be the case for two equal receivers. We therefore define \( \text{PSD}_{n_{\text{corr}}} = \rho N_0 \) and \( \text{PSD}_{n_1} = \text{PSD}_{n_2} = (1 - \rho) N_0 \), where \( \rho \) denotes the noise correlation between the two receivers. When \( \rho = 1 \), the noise is fully correlated, and fig. 1a simplifies to fig. 1b. When \( \rho = 0 \), the noise in one receiver is completely uncorrelated with the noise in the other receiver.

Without loss of generality, we define \( N_0 \) = 1 in the remainder of this paper.

A. Relation with multi-antenna systems

Multi-antenna systems already have multiple receivers available, so for these systems the crosscorrelation process can be implemented with very low overhead cost. Multi-antenna systems are not only considered more and more in recent literature for CR communications, but also for spectrum sensing [11]–[13], [15], [17]. Nevertheless, we will show in section VI-B that crosscorrelation can dramatically reduce measurement time for a single-antenna system, as well as allow the system to be much more linear.

The model shown in fig. 1a is an abstraction that can cover three possible implementations, shown in fig. 2:

- A one-antenna system, where the output of the antenna is processed by two receivers. Noise correlation is introduced by the antenna noise, shared lossy, and hence noisy components, and crosstalk between the two receivers, see fig. 2a.
- A two-antenna system of which only one antenna is used during spectrum sensing, in a similar way as in the previous point, see fig. 2b.
- A two-antenna system of which both antennas are used during spectrum sensing, each with its own receiver. It is often assumed that the noise samples obtained through different antennas are independent, but mutual coupling and spatial noise correlation will introduce noise correlation in the two receivers [14], see fig. 2c.

A generalization to more than two antennas is not straightforward. Since crosscorrelation multiplies the outputs of two receivers, the result is already in the power domain. Multiplying the outputs of more than two receivers results in something different from energy detection, which may still be useful, but is outside the scope of this work. It may be possible to combine the outputs of pairwise correlations of the different antennas in some clever way, such as is done in [13]. Irrespective of whether further generalization is possible, the simplest implementation deals with two antennas, and quantifying the limits of this system is useful.

For crosscorrelation in fading and mobile environments, the phase difference of the received signal between two antennas may change arbitrarily during the spectrum sensing process [18]. This will cause signal decorrelation, and is not covered in the analysis. Square-law combining in diversity reception receivers, as has also been proposed in literature [12], avoids this problem by autocorrelation at each receiver, but as a downside suffers from the same SNR-wall problems as regular energy detection with one receiver, as it only decreases the variance of the estimates, and not the bias caused by noise uncertainty. Other diversity approaches, such as maximum ratio combining and selection combining, first require some form of SNR-estimation, so they are not directly applicable to spectrum sensing in our context.

IV. SNR-Wall for Crosscorrelation

Based on the model discussed in section III, we can start to find an expression for the SNR-wall for an arbitrary noise correlation \( \rho \). Starting with fig. 1a, we find for \( y[N] \)

\[
\begin{align*}
y_{\text{re}}[N] &= \frac{1}{N} \sum_{1}^{N} \left( x_{\text{re}}(n_1, \text{re} + n_2, \text{re}) + x_{\text{im}}(n_1, \text{im} + n_2, \text{im}) 
\quad + n_{1, \text{re}} n_{2, \text{re}} + n_{1, \text{im}} n_{2, \text{im}} + x_{\text{re}}^2 + x_{\text{im}}^2 \right) 
\text{signal power} \\
y_{\text{im}}[N] &= \frac{1}{N} \sum_{1}^{N} \left( x_{\text{re}}(n_1, \text{im} - n_2, \text{im}) + x_{\text{im}}(n_2, \text{re} - n_1, \text{re}) 
\quad + n_{1, \text{im}} n_{2, \text{re}} - n_{1, \text{re}} n_{2, \text{im}} \right)
\end{align*}
\]

(1)

where the subscripts “re” and “im” denote the real and imaginary parts. Define \( Y \) as the stochastic variable representing \( y[N] \), and \( Y_{\text{re}} \) and \( Y_{\text{im}} \) as the real and imaginary part of \( Y \), respectively, i.e., \( Y = Y_{\text{re}} + j Y_{\text{im}} \). For notational convenience, we leave out the parameter \( N \) from \( Y_{\text{re}} \) and \( Y_{\text{im}} \).

We assume that we have no oversampling; for a complex process, this means a sample rate \( f_s = W \) for both the real and imaginary components. As these samples are i.i.d., the central limit theorem states that their average converges to a normal distribution for a large number of samples. Hence, the outputs \( Y_{\text{re}} \) and \( Y_{\text{im}} \) will tend to a normal distribution for long averaging time.

Some papers, including [3], [9], assume an analog implementation. To a good approximation, one can simply substitute summations by integrals and \( N = TW \) to obtain results for an analog implementation, where \( T \) is the measurement time [9].

Before we continue, it is important to remark that there may be some differences in the two receivers, such as phase, gain, time and frequency offsets, unequal noise floors and unequal filters. These factors will have their influence on the
A. Case I: no phase offset

When we assume that both receivers are perfectly in phase, i.e., \( \Delta \phi = 0 \), we can simply discard \( Y_m \), because it does not contain signal power. This means we can base our decision entirely on \( Y_c \), of which the first two central moments are derived in [19, p378] based on the assumption that \( N \) is large enough. For our model, this results in

\[
E[Y_c] \approx \rho + \text{SNR}
\]

\[
\text{var}[Y_c] \approx \frac{1}{2N} (2\text{SNR}^2 + (2 + 2\rho)\text{SNR} + 1 + \rho^2).
\]

There are two hypotheses between which the system has to decide:

\( H_0 \): no signal present;

\( H_1 \): signal present.

Associated with these hypotheses are a mean and variance as defined by eq. (2). For \( H_0 \) (SNR = 0)

\[
\mu_0 = \rho
\]

\[
\sigma_0^2 = \frac{1 + \rho^2}{2N}
\]

and for \( H_1 \) (SNR > 0)

\[
\mu_1 = \rho + \text{SNR}
\]

\[
\sigma_1^2 = \frac{1}{2N} (2\text{SNR}^2 + (2 + 2\rho)\text{SNR} + 1 + \rho^2).
\]

The probability of false alarm \( P_{FA} \) is the probability the system wrongly decides \( H_1 \):

\[
P_{FA} \equiv \mathbb{P}(H_1 | H_0) = Q\left(\frac{K - \mu_0}{\sigma_0}\right)
\]

where \( K \) is the detection threshold and \( Q(\cdot) \) is defined as

\[
Q(x) \equiv \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-z^2/2} \, dz.
\]

The probability of detection \( P_D \) is the probability the system correctly decides \( H_1 \):

\[
P_D \equiv \mathbb{P}(H_1 | H_1) = Q\left(\frac{K - \mu_1}{\sigma_1}\right).
\]

The probability of missed detection \( P_{MD} \) is given by \( P_{MD} \equiv \mathbb{P}(H_0 | H_1) = 1 - P_D \).

Setting the threshold \( K \) is the key problem in energy detection, and is based on the noise power. Suppose \( N_0 \) is estimated by \( \hat{N}_0 \), with any distribution of \( \hat{N}_0/N_0 \), such that

\[
(1 - \epsilon_1) \leq \hat{N}_0/N_0 \leq (1 + \epsilon_2)
\]

with \( 0 \leq \epsilon_1 < 1 \) and \( \epsilon_2 \geq 0 \). To guarantee a certain detection performance, \( P_{FA} \) should be upperbounded, which means \( K \) should be purposely biased such that the worst-case estimation error still achieves the required \( P_{FA} \), i.e., by multiplying the threshold by a factor \((1 - \epsilon_1)^{-1} \). As a consequence (eq. (8)), \( P_{FA} \) will be lowered. The worst-case \( P_D \) occurs when \( \hat{N}_0/N_0 = (1 + \epsilon_2) \), so when guaranteeing a certain desired \( P_D \), we can take this worst-case \( P_D \) as our desired \( P_D \). This can be done elegantly by defining the peak-to-peak uncertainty \( U \), which affects both the correlated and uncorrelated parts of the noise, as [3]

\[
U \equiv \frac{1 + \epsilon_2}{1 - \epsilon_1}
\]

and setting \( K \) to [3]

\[
K = U(\mu_0 + \sigma_0 Q^{-1}(P_{FA,des})).
\]

We have plotted the theoretical and simulated \( P_{MD} \) and \( P_{FA} \) for \( U = 1 \) (noise power exactly known) in fig. 3, showing that the theory matches the simulations very well.

A subtle point in the above discussion, which does not influence the remainder of this paper, but does require some explanation, is that if the support of \( \hat{N}_0/N_0 \) is \((0, \infty) \) (which would be the case for example when \( \hat{N}_0/N_0 \) has a log-normal distribution), \( \epsilon_1 = 1 \) and \( \epsilon_2 = \infty \), and thus \( U \) goes to infinity, making detection impossible. It can be intuitively understood, that when \( \mathbb{P}(\hat{N}_0/N_0 \leq 1 - \epsilon_1) \ll P_{FA,des} \) and \( \mathbb{P}(\hat{N}_0/N_0 \geq 1 + \epsilon_2) \ll 1 - P_{D,des} \) it will not have a significant impact on the obtained \( P_{FA} \) and \( P_D \). Note that this
implies that $U$ becomes a function of $P_{\text{FA,des}}$ and $P_{\text{D,des}}$. A more elaborate and formal discussion is given in [3].

Equations (4) to (6), (8) and (11) do not directly reveal the SNR-wall, so we proceed to find simpler expressions. Filling in eq. (11) into eq. (8), and approximating for small SNR (SNR $\ll 1$) and long averaging ($N \gg 1$), we find

$$P_{\text{b}} \approx Q \left( \frac{\rho(U - 1) - \text{SNR}}{\sqrt{N} + U \sqrt{\kappa N}} \right)$$

where $\kappa \equiv \frac{1}{2} + \frac{1}{2} \rho^2$. Given also a desired probability of detection $P_{\text{D,des}}$, we can solve eq. (12) for the minimum SNR, SNR$_{\text{min}}$, necessary to obtain both $P_{\text{FA}} \leq P_{\text{FA,des}}$ and $P_{\text{D}} \geq P_{\text{D,des}}$:

$$\text{SNR}_{\text{min}} \approx \rho(U - 1) + \frac{\kappa U B}{\sqrt{N}} + \frac{A^2 \rho + 1}{N} - \frac{\sqrt{N(\kappa + (U - 1)(\rho^2 + \rho))}}{2}$$

where $\lambda \equiv (\rho + 1)^2 A^2 + \kappa U B (\rho + 1) \sqrt{N}$, $A \equiv Q^{-1}(P_{\text{D,des}})$, and $B \equiv Q^{-1}(P_{\text{FA,des}})$. Simplifying this using $N \gg 1$ and $|A|, |B|$ fairly small for reasonable detection probabilities ($|A|, |B| \leq 10$ gives $P_{\text{FA,des}}, P_{\text{MD,des}} \geq 10^{-23}$) we find

$$\text{SNR}_{\text{min}} \approx \rho(U - 1) + \frac{\kappa U B - A \sqrt{\rho^2 + \rho}(U - 1) + \kappa}{\sqrt{N}}$$

which for $\rho = 1$ (standard energy detection) simplifies to the result derived in [3] for autocorrelation.

By substituting $N = T W$ in eq. (14) and solving for $T$ we find a convenient equation to get a first-order estimate of the minimum required measurement time $T_{\text{min}}$:

$$T_{\text{min}} \approx \frac{1}{W} \left( \frac{\kappa U B - A \sqrt{\rho^2 + \rho}(U - 1) + \kappa}{\text{SNR}_{\text{min}} - \rho(U - 1)} \right)^2$$

provided $\text{SNR}_{\text{min}} > \rho(U - 1)$.

For infinitely long averaging, we see a minimum SNR below which we cannot detect a signal, the SNR-wall:

$$\text{SNR}_{\text{wall}} = \lim_{N \to \infty} \text{SNR}_{\text{min}} = \rho(U - 1).$$

Note that this result is independent of $P_{\text{FA,des}}$ and $P_{\text{D,des}}$. Clearly, when $\rho = 0$, there is no SNR-wall, independent of noise uncertainty $U$.

SNR$_{\text{min}}$ according to eq. (14) is plotted in fig. 4 for several noise correlations $\rho$ and uncertainties $U$ with $P_{\text{FA}} = P_{\text{MD}} = 10^{-5}$ (similar results can be obtained for other $P_{\text{FA}}$ and/or $P_{\text{MD}}$ as SNR$_{\text{wall}}$ does not depend on $P_{\text{FA,des}}$ and $P_{\text{D,des}}$). For $U = 1 \text{dB}$, which is a typical value for the noise uncertainty of receivers [20], and $\rho = 1$, SNR$_{\text{wall}} \approx -6 \text{dB}$, which is still about 9 dB off from the sensing limit required by IEEE 802.22 (this will be elaborated in section V). From fig. 4, one can clearly observe that both a lower uncertainty $U$ and a lower correlation $\rho$ lower the SNR-wall, which allows smaller signals to be detected.

The theoretical (based on eq. (8)) and simulated $P_{\text{MD}}$ is plotted in fig. 5 for different $U$ and $\rho$, with $P_{\text{FA,des}} = 0.1$. The threshold is set according to eq. (11). The simulations assume worst-case noise estimates for $P_{\text{MD}}$, such that the simulated $P_{\text{MD}}$ should coincide with eq. (8). It shows that for higher $U$, a lower $\rho$ is required to keep the detector robust, which is consistent with our results obtained using approximations.

B. Case II: phase offset

We have shown that, without phase offset, crosscorrelation reduces the SNR-wall. When there is phase offset $\Delta \phi$ between the two receivers, e.g., due to a timing difference in the mixer, we find $Y' = e^{j \Delta \phi} Y$, where $Y$ denotes the situation without phase offset and $Y'$ the situation with phase offset. When $\Delta \phi$ is negligibly small, or can be estimated and corrected for, there is no problem, and we can use the approach of the previous subsection. Here, we are interested in finding a solution for arbitrary $\Delta \phi$. Using the statistic $|Y'|$ makes the result independent of phase offset [21].

It is important to note that now we use both $Y_{\text{re}}$ and $Y_{\text{im}}$. Because this introduces more noise, it has a detrimental effect
on the detection performance and should therefore be avoided when possible. Hence, the situation where we cannot avoid it warrants a new derivation. Although the exact statistics for this situation have been calculated in [21], we use another approach as it yields more tractable results, allowing us to find the SNR-wall. Again we assume that \( N \) is large enough to justify Gaussian approximations for \( Y_{re} \) and \( Y_{im} \).

When \( \rho = 0 \) and SNR = 0 (or, equivalently, \( \mu_0 = 0 \)), \( Y_{re} \) and \( Y_{im} \) have equal variance. The covariance between \( Y_{re} \) and \( Y_{im} \), \( E[(Y_{re} - E[Y_{re}])(Y_{im} - E[Y_{im}])] \), is easily found to be 0 due to the model assumption that \( N_{1,re} \), \( N_{1,im} \), \( N_{2,re} \) and \( N_{2,im} \) are mutually independent. Hence, \( Y_{re} \) and \( Y_{im} \) are uncorrelated, and, because both are normally distributed, independent. Thus, \( |Y| = \sqrt{Y_{re}^2 + Y_{im}^2} \) is Rayleigh-distributed, giving \( P_{FA} = \exp(-K^2/2\sigma_0^2) \). Solving for the threshold \( K \) and incorporating the uncertainty gives

\[
K = U \sigma_0 \sqrt{2 \ln \frac{1}{P_{FA,des}}}. \tag{17}
\]

For other situations, i.e., \( \rho > 0 \) and/or SNR > 0, the first two raw moments are derived in [16]. These moments can be converted to central moments, which, for our system model, results in:

\[
\begin{align*}
\mu_0' &= \sqrt{\rho^2 + \frac{\beta}{N}(1 - \rho^2)} \\
\mu_1' &= (\text{SNR} + \rho)^2 + \frac{\beta}{N}((1 - \rho^2) + 2(1 - \rho)\text{SNR}) \\
\sigma_0^2 &= \frac{1}{N}(1 - \beta + \beta \rho^2) \\
\sigma_1^2 &= \frac{1}{N}(\text{SNR}^2 + (2 - 2\beta + 2\beta \rho)\text{SNR} + 1 - \beta + \beta \rho^2) \tag{18}
\end{align*}
\]

where \( \beta \) is an interpolation function, defined as

\[
\beta = \frac{\pi}{4} \frac{1}{N} \left( \frac{\Gamma(N + \frac{1}{2})}{\Gamma(N)} \right)^2 \left( 1 - \frac{(\rho + \text{SNR})^2}{E[|Y|^2]} \right) + \frac{1}{2} \frac{(\rho + \text{SNR})^2}{E[|Y|^2]} \tag{19}
\]

with \( \Gamma(x) \) the mathematical Gamma function, and \( E[|Y|^2] = \sigma_0^2 + \mu_0^2 \) (or, when SNR = 0, \( E[|Y|^2] = \sigma_0^2 + \mu_0^2 \)). Note that we use an accent to indicate that these central moments are different from the central moments defined in eqs. (4) and (5). \( \beta \) varies between \( \frac{1}{2} \) and \( \frac{3}{4} \), where \( \beta = \frac{1}{2} \) when the signal and/or correlated noise dominates, and \( \beta = \frac{3}{4} \) when the uncorrelated noise dominates [16].

As a verification, suppose the signal and correlated noise dominate the uncorrelated noise, so \( \beta = \frac{3}{4} \). When there is no phase offset, it follows from eq. (1) that \( \text{var}[Y_{re}] \gg \text{var}[Y_{im}] \), and hence, \( \text{var}[|Y|^2] \approx \text{var}[Y_{re}] \). Noting that \( |Y| = |Y'| \), we should have \( \text{var}[|Y'|] \approx \text{var}[Y_{re}] \). Indeed, for \( \beta = \frac{3}{4} \) we find \( \sigma_0^2 \) and \( \sigma_1^2 \) in eq. (18) to be equal to \( \sigma_0^2 \) and \( \sigma_1^2 \), respectively, in eqs. (4) and (5).

For \( \mu_0' \) such that \( \lim_{N \to \infty} \mu_0' > 0 \), and sufficiently long measurement time, the distribution of \( |Y| \) will tend to a Gaussian distribution, and is hence adequately described by the first two moments [21]. So we can set the decision threshold again as in eq. (11), substituting \( \mu_0 \) and \( \sigma_0 \) by \( \mu_0' \) and \( \sigma_0' \), respectively:

\[
K = U(\mu_0' + \sigma_0' Q^{-1}(P_{FA,des})). \tag{20}
\]

Using eq. (18), we have plotted the theoretical and simulated \( P_{MD} \) and \( P_{FA} \) in fig. 6 for \( U = 1 \). Theory matches simulations very well. The slightly increased \( P_{FA} \) for low \( \rho \) is caused by the fact that the pdf is close to a Rayleigh-distribution, and hence has a heavier tail than what is estimated using the Gaussian approximation.

In both cases (eqs. (17) and (20)) we find \( K = \mu_0' + \sigma_0' F(P_{FA,des}) \), with \( F(x) \) some function of \( x \). This means we can repeat the derivation as shown in the case without phase offset from eq. (8) to eq. (14). With the central moments of eq. (18) we find \( \text{SNR}_{\text{rms}} \) as defined in eq. (14) with \( \kappa \) replaced by \( 1 - \beta + \beta \rho \). Hence, we arrive at the same result for the SNR-wall in eq. (16):

\[
\text{SNR}_{\text{wall}} = \rho(U - 1). \tag{21}
\]

For completeness, we have also plotted the theoretical and simulated \( P_{MD} \) for the situation with phase offset and noise uncertainty by taking the absolute value \( |Y| \), see fig. 7. We have used the central moments of eq. (18) and worst-case noise estimates for \( P_{MD} \), and, as can be observed, theory matches simulations very well. Although the SNR-wall is the same, taking the absolute value of the output increases the variance, and therefore the measurement time needs to be longer for a certain \( P_{FA,des} \) and \( P_{D,des} \). This is clearly visible for \( \rho = 0 \) in fig. 7b as compared with fig. 5b: for \( P_{MD} = 10^{-3} \) the required measurement time is roughly doubled.
to the noise uncertainty (visible as a non-white noise floor), it is clearly present in the crosscorrelation spectrum.

VI. DISCUSSION

Tandra and Sahai [1] suggested three options to get around the SNR-wall for energy detection: 1) impose some structure on the primary signal (i.e., use knowledge of the signal to be detected), 2) force some diversity in fading to improve SNR (e.g., by moving around while sensing), or 3) “somehow reduce the noise uncertainty”. Equation (16) shows that crosscorrelation achieves the last mentioned option: the effective noise uncertainty $\rho(U - 1)$, and therefore the SNR-wall, can be reduced by minimizing $\rho$.

In the previous paragraph we argued that the SNR-wall is reduced by crosscorrelation. On the other hand, we know that crosscorrelation removes uncorrelated noise, hence improving the SNR [7]. So, one could also argue that the SNR-wall remains the same, and the SNR is improved by crosscorrelation. For example, when $\rho = 0.01$, only 1% of the noise power will eventually remain, improving the effective SNR by 20 dB. Both arguments lead to the same conclusion: the minimum signal power that can be detected is improved by 20 dB.

Generalizing, we can translate the SNR-wall of eq. (16) to the sensitivity limit of a crosscorrelation receiver (i.e., the minimum signal power that can be detected for infinite measurement time), to obtain

$$P_{\text{sn.min}} = -174 + \text{NF} + 10 \log_{10} W + 10 \log_{10}(\rho(U - 1)) \text{ dBm}$$

(22)

A. Other types of noise

Equations (2) and (18) were derived based on explicit stationary white Gaussian noise assumptions. For signals that do not occupy a certain band for 100% of the time, such as used in GSM and WiMAX, the signal is not stationary. The nonstationary behavior does change the statistics, because products of signal with noise components will sometimes occur and sometimes not, so a new derivation will be required to properly set the threshold for given $P_{\text{FA,des}}$ and $P_{\text{D,des}}$. Intuitively, as crosscorrelation is a form of energy detection, it will detect the average energy of the signal. Simulations indicate that the equations derived in this paper do give a
very good approximation for these ‘duty-cycled’ signals, see the DCWG line in fig. 9. Therefore, since SNR\textsubscript{wall} does not depend on \( P_{\text{FA,des}} \), \( P_{\text{D,des}} \) and \( N \), we expect to obtain the same SNR\textsubscript{wall} for this type of signals, where the SNR should be interpreted as the average SNR over time.

The statistics will also change when the whiteness and/or the Gaussianity of the noise are changed. The central limit theorem (CLT) states that the distribution of the output of the crosscorrelation process will converge to a Gaussian distribution, regardless of the input distributions. This means, that noise reduction will be obtained for any kind of noise source, at the same asymptotic rates. However, the initial expected value and variance will depend on the type of noise, as well as the convergence to the asymptotic rates, which will have an impact on the detection process.

A practically relevant example is quantization noise from the ADC, which can be conveniently modeled as a white noise source with a uniform pdf of its amplitude between \(-\text{LSB}/2\) and \(\text{LSB}/2\). A simulation has been performed where the signal and all noise sources in fig. 1 have been replaced by white uniformly distributed sources, see the WU line in fig. 9. With the signal being present, the \( \mu \) and \( \sigma^2 \) found for the WG-simulation matches \( \mu_1 \) and \( \sigma^2_1 \) of eq. (5). For the WU-simulation, the simulated \( \mu \) and \( \sigma^2 \) are very close to \( \mu \) and \( \sigma^2 \) for white Gaussian sources, as can be seen in fig. 9 (only \( \sigma^2 \) shown). By using the threshold calculated for white Gaussian sources, \( P_{\text{FA}} \) and \( P_{\text{MD}} \) for white uniform sources are found to be almost identical to those for white Gaussian sources, indicating that our results are also relevant for uniformly distributed noise.

Non-white noise, such as would be the case for phase noise (partly white, partly \(1/f^2\) dependency) and flicker noise (\(1/f\) dependency), seem to give similar \( \mu \) but quite different \( \sigma^2 \) as compared to the WG-simulation. The simulated variance for PN (phase noise) is shown in fig. 9, where the signal is still white Gaussian. We believe the reason for the higher \( \sigma^2 \) is that now the samples in each receiver are correlated in time, which limits the reduction in variance. This is also a known effect for standard energy detection [22]. As a verification we have also simulated bandlimited Gaussian noise with \( L = 10 \) times oversampling (such that it covers the range from \(0\) to \(f_s/L\)), see the OG line in fig. 9. When the threshold is calculated using \( N/L \) samples instead of \( N \) samples, the desired \( P_{\text{FA}} \) and \( P_{\text{D}} \) can be found, which is in accordance with [22]. We leave further analysis of these effects as future work.

### B. Practical considerations

Just like the noise level estimation, the correlation factor has to be estimated as well [8]. Even with extreme care in the design of the two receivers to minimize coupling, there will be some correlation of the noise, such as black-body radiation from surrounding objects, coupling between antennas, or noise-like interference. Hence, it seems impossible to obtain \( \rho = 0 \). Moreover, the assumptions of Gaussianity and stationarity of the noise may fail for long measurement times, so a larger part of the uncertainty may be residing in the correlated noise, reducing the improvement provided by the crosscorrelation technique. Thus, further work is needed to find the actual practical achievable limits.

With respect to crosscorrelation, at least two systems have already been published. In [23], two receivers were used to measure the noise of a single resistor. The sensitivity with respect to a single measurement path is improved by 50 dB, corresponding to \( \rho = 10^{-5} \). This low noise correlation is achieved by directly connecting two LNAs to the device-under-test (DUT), minimizing coupling between the channels. The noise coupling is a lot higher in [7] due to shared components. The NF of that system was reduced from 24 dB to 4 dB, a reduction of 20 dB, corresponding to \( \rho = 10^{-2} \), but, according to eq. (22), not good enough to comply with 802.22 regulations. The important advantage of crosscorrelation in [7] is that it allows signal attenuation to obtain higher linearity, while the same noise floor is obtained after crosscorrelation. These systems indicate that crosscorrelation can significantly improve the ability of spectrum sensing devices to detect small signals, but also that crosscorrelation allows a trade-off between NF and linearity, without affecting the SNR-wall and with a known penalty on measurement time, both of which are very important considerations for energy detection.

When a system does not have multiple receivers available, these improvements may be enough to warrant the use of an additional receiver, solely used for spectrum sensing. This is clearly illustrated by fig. 10, which shows the required number of samples (per receiver) for detection of a signal for different SNR as a function of \( \rho \) for \( U = 1\) dB. For example, for SNR = \(-4\) dB, crosscorrelation with \( \rho = 0.1 \) reduces measurement time by a factor of 13 as compared to \( \rho = 1 \), which not only allows more time to use the spectrum for data transmission, but also reduces the total energy consumption of the spectrum sensing process by more than a factor 6. This more than makes up for the additional power temporarily required by turning on the second receiver. When the higher sensitivity of crosscorrelation is not required, it can be turned off or used to lower the variance in the energy detection process by acting as a second energy detector.
VII. CONCLUSIONS

Energy detection by autocorrelation is a technique for spectrum sensing that requires no knowledge of the signal to be detected. Due to uncertainty in the noise level it suffers from an SNR-wall: a minimum SNR below which reliable detection is impossible. Crosscorrelation is a generalized technique for energy detection, which is based on two paths processing the same input signal, such that the noise added in one path is uncorrelated with the noise in the other path. In combination with a passive attenuator, where receiver NF is traded for linearity, the technique allows for reliable detection of weak signals even in the presence of much stronger signals.

In this paper we have derived equations that relate the detection performance and relevant system parameters of a crosscorrelation system. We have also obtained an explicit expression to estimate the required measurement time to obtain a desired detection performance. Close to the SNR-wall for standard energy detection, crosscorrelation can offer orders of magnitude faster detection. In an example we have shown that for realistic system parameters and noise uncertainty, a crosscorrelation system will be able to satisfy the stringent IEEE 802.22 requirements for spectrum sensing when $\rho \leq 0.125$, without having any knowledge of the signals to be detected.

By using approximations of these expressions, we have shown that the SNR-wall is directly proportional to the degree of noise correlation, which can be very low as shown by existing implementations. This suggests that crosscorrelation can provide a significant improvement in detection capabilities for a device that uses energy detection for spectrum sensing.

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REFERENCES

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