Accounting for Inpatient Wards When Developing Master Surgical Schedules

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BACKGROUND: As the demand for health care services increases, the need to improve patient flow between departments has likewise increased. Understanding how the master surgical schedule (MSS) affects the inpatient wards and exploiting this relationship can lead to a decrease in surgery cancellations, a more balanced workload, and an improvement in resource utilization. We modeled this relationship and used the model to evaluate and select a new MSS for a hospital.

METHODS: An operational research model was used in combination with staff input to develop a new MSS. A series of MSSs were proposed by staff, evaluated by the model, and then scrutinized by staff. Through iterative modifications of the MSS proposals (i.e., the assigned operating time of specialties), insight is obtained into the number, type, and timing of ward admissions, and how these affect ward occupancy.

RESULTS: After evaluating and discussing a number of proposals, a new MSS was chosen that was acceptable to operating room staff and that balanced the ward occupancy. After implementing the new MSS, a review of the bed-use statistics showed it was achieving a balanced ward occupancy. The model described in this article gave the hospital the ability to quantify the concerns of multiple departments, thereby providing a platform from which a new MSS could be negotiated.

CONCLUSION: The model, used in combination with staff input, supported an otherwise subjective discussion with quantitative analysis. The work in this article, and in particular the model, is readily repeatable in other hospitals and relies only on readily available data. (Anesth Analg 2011;112:1472–9)

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OR block of capacity typically represents a full day of OR time. This assignment of the surgical specialties to each OR block is referred to as the MSS.1 Once developed, the MSS is simply repeated throughout the year. At NKI-AVL, the MSS represents a 1-week period in which the surgeons from the 6 surgical specialties are assigned to 1 or more of the 5 operating days. An example MSS for 5 ORs is shown in Figure 1.

Developing a good MSS can be a complicated task.2 There are many restrictions within the department of surgery, such as limited specialized equipment and physician availability, which need to be adhered to. These restrictions are, for the most part, known in advance. However, the relationship between the MSS and the ward is much less direct and plagued with uncertainties. For example, it may be known that every patient from a given specialty will be admitted to the ward after surgery; however, the number of patients who receive surgery during 1 OR block changes from week to week and it is not known with certainty how long each patient will stay. Because of this uncertainty, MSSs are often developed without explicit consideration for the wards. Notable exceptions (discussed in Vanberkel et al.3) used models from operations research to define and emulate the relationship between the ORs and the ward.

The use of industrial engineering and operations research in health care4–8 is not new. However, authors have noted that the implementation rates in health care settings has lagged behind that of other industries.9–11 More specifically, as is related to the development of MSSs, the literature is replete with studies12–15 that use complex mathematics to derive optimal MSSs; however, the rate of implementation for this type of study is unknown but

Netherlands Cancer Institute–Antoni van Leeuwenhoek Hospital (NKI-AVL) is a comprehensive cancer center that provides hospital care and research and is located in Amsterdam, The Netherlands. The hospital has 150 inpatient beds and the outpatient clinic receives approximately 24,000 appointment requests every year, making it approximately the size of a mid-sized general hospital. As with many Dutch hospitals, NKI-AVL is eager to improve access and increase capacity. To this end, the hospital has expanded its operating capacity from 5 to 6 operating rooms (ORs). The hospital welcomed this expansion as an opportunity to develop a new master surgical schedule (MSS). Developing a good MSS is a complex task that requires the commitments and concessions of many competing stakeholders.

Typically, and as is the case at NKI-AVL, the yearly amount of operating time allotted to each specialty reflects patient demand and hospital priorities. To implement a chosen allotment, and to make the surgical department more manageable, many hospitals divide their surgical capacity into OR blocks over a certain planning period. One
presumed low. To overcome this, Cardoen et al. encourage the provision of additional information on the behavioral factors that coincide with the actual implementation. Identifying the causes of failure or the reasons that lead to success, may be of great value to the research community.

In this article, we describe the process of developing a new MSS for NKI-AVL and the first results observed after its implementation. The development process, which combined an operational research model and staff input, led to an MSS that was agreeable to staff from both the wards and the OR. Staff selected and implemented an MSS that the model predicted would result in a balanced ward occupancy.

METHODS

The development of the new MSS was completed over 3 months in an iterative manner. A team was formed consisting of a team leader from the surgical department, a team leader from the inpatient wards, the manager of both groups, and 2 of this article’s authors who work in the hospital’s operations improvement group. The team members from the surgical department ensured MSS proposals did not cause conflicts within the OR, such as with physician schedules and available equipment. The projected impact that each MSS proposal would have on the wards was evaluated with the operational research model described below. Each new MSS proposal represented a new scenario to be evaluated by the model. From the model output, staff decided whether the MSS was acceptable or if further modifications to the MSS were necessary.

The original MSS was roughly developed as follows. Based on production targets, the number of OR blocks to be assigned to each specialty during the 1-week MSS cycle was determined. Next, the physicians’ commitments elsewhere in the hospital were determined and their preferred operating days were considered. Potential equipment and resource conflicts were addressed; for example, it would be problematic to assign 2 specialties to the same operating day when both routinely require the same specialized OR. Considering these restrictions, OR staff proposed the original MSS.

To determine how the original MSS affected the wards, the operational research model described in the following subsection was used. As illustrated in the Results section, the original MSS results in an unbalanced ward occupancy (the motivation for this metric is also provided in the Results section). As such, the team decided the original MSS was not acceptable.

Next, modifications to the original MSS were made and a new MSS proposal was put forth. Given that in this project we were not asking surgical specialties to change how they operate (e.g., the number of procedures they perform in an OR block and/or the invasiveness of their procedures, which can dictate length of stay), modifications to the MSS were limited to changes in the assignment of surgery specialties to OR blocks. Essentially, modifications consisted of swapping a specialty operating on one day with a specialty operating on a different day. Deciding which blocks to swap followed at first, from OR staff knowledge of what was possible within the constraints of the OR, and second, by intuition gained from seeing results from several MSS proposals. Figure 2 illustrates the type of modifications made.

A number of MSSs were proposed, and the impact that each would have on the ward was evaluated by the model. This process of modifying and evaluating MSSs continued until an MSS was found that satisfied staff from both the OR and the wards. A schematic overview for this process is displayed in Figure 3.
Surgical Schedules and Inpatient Wards

The MSS chosen by the team was implemented concurrently with the opening of the new OR in March of 2009. The new OR was phased in over several months, and once it became fully utilized, ward occupancy statistics were collected. The data, observed over a 33-week period when all 6 ORs were being regularly scheduled, were compared with what was projected from the model. The purpose, to ensure a more balanced ward occupancy, was indeed achieved with the implemented MSS. In this way, we could validate the model output and confirm that the implemented MSS was resulting in the desired ward occupancy.

Model Description

The intuition for the model follows from the intended flow of surgical patients from the OR to the ward, and finally, to being discharged. Using 12 months of historical data from the year 2008, we determined, for each surgical specialty, the probabilities for 1, 2, 3, … inpatient procedures being completed in 1 OR block. These numbers express the number of patients admitted to the ward designated to an OR block of that specialty. Every day after surgery, each of these patients has a probability of being discharged. These discharge probabilities are specific for each surgical specialty and for each day after surgery. They were likewise derived from the historical data.

From probability theory it is known that when multiple experiments have 2 (and only 2) possible outcomes, then the probability for the number of experiments resulting in each outcome can be computed with a binomial distribution (assuming experiments are independent and identically distributed). For example, say we know that given the choice between a vegetarian and a nonvegetarian dinner, 20% of people will choose vegetarian and the other 80% will choose nonvegetarian. Say that 200 people arrive to dinner, then the binomial distribution can tell us the probability that exactly 60 (or any number <200) vegetarian dinners will be requested. The “experiment” in this case is the person’s choice and the “outcomes” are vegetarian and nonvegetarian dinners. It is easy to imagine the value of such information to a caterer.

In our case, consider the patient as the experiment with 2 outcomes, “stay” or “be discharged.” From the data above, we know the probability of being discharged and conversely the probability of staying. If we know the number of inpatients hospitalized today (i.e., the number of experiments), then we can use the binomial distribution to compute the number of discharges and conversely we know the number of beds needed tomorrow.

The inherent assumption of using the binomial distribution for this purpose is that all patients (experiments) have equal probability of each outcome and that the outcome is independent of other patients, i.e., it is assumed that the patients are identically distributed and independent. The independence assumption is natural because it implies that the amount of time one patient is in the hospital does not influence the amount of time another patient is in the hospital. The identically distributed requirement means that we must compute the number of beds needed tomorrow (and the number of cases completed in 1 OR block), for all identically distributed cohorts of patients separately. In other words, the parameters of the binomial distribution must reflect all of the patients in a given cohort (for a discussion on defining statistically equivalent patient cohorts, see the article by Harper17). Finally, because patients are independent, the total number of beds needed tomorrow is computed by adding the beds needed by each cohort. Because we are adding distributions and not the mean, the additions are computed using discrete convolutions (we provided a formal model description in the Appendix).

Because this project considers tactical planning (planning decisions made over the intermediate planning horizon and without complete knowledge of what is to come), a certain level of aggregation is required. When an MSS is being developed, not all of the details about the planning horizon are known, i.e., which patients will arrive, which doctors will be available, etc. What is known is that a certain volume of patients with a certain case mix is expected. Hence, the assignment of surgical specialties (not individual patients) to OR blocks is the primary factor controlled by the OR planners. We reflect this in our model by aggregating patient data such that each surgical specialty is a single patient cohort.

To use the binomial distribution as discussed above, the patients within each surgical specialty should not be too inhomogeneous. If a somewhat inhomogeneous population is grouped together, this causes the ward census distribution to have longer tails (although the mean remains the same) and will overestimate the bed requirements when staffing for a certain percentile of demand. Conversely, however, less aggregation (such as dividing a specialty by short- and long-stay patients) decreases the sample size from which to derive the parameters, which in turn reduces the statistical confidence of the estimated parameters. In our case, aggregating the data by specialty allowed for enough data to have a sufficient sample size and resulted in relatively homogeneous patient cohorts. As discussed in the following paragraph, in nonspecialty hospitals where a certain level of aggregation is required, the model can still be used.

In cases in which patients of a surgical specialty are too inhomogeneous to be aggregated into a single cohort, the model can still be used. First, the inhomogeneous specialty has to be divided into multiple homogeneous cohorts and...
then these cohorts can be treated as if they were assigned their own OR block. Using this, the binomial distribution is applied as described above to determine the bed requirements of each cohort. Again, using the independence assumptions, these cohorts can be added (with discrete convolutions) to determine the total bed requirements for the complete surgical specialty.

Historical records indicate the number of inpatient procedures completed by a specialty assigned to an OR block. This provides the probability of the number of beds needed on the day of surgery (i.e., the number of experiments). With this information, and using the binomial distribution, we can determine the probability for the number of beds needed each day after surgery. In this way, the model creates an explicit relationship between each OR block and the inpatient wards. Finally, by including all OR blocks and by taking care to account for patients overlapping from one MSS cycle to the next, the overall need for beds for a given MSS can be computed. The formal model is outlined in the Appendix and is described in detail in Vanberkel et al.18

The model is depicted graphically for a single OR block in Figure 4.

Although it is changing, health care management is notorious for making planning decisions based on averages.19 However, if the caterer in our example would only prepare the average amount of vegetarian meals expected from the crowd of 200, a shortage would be realized 46% of the time. In the same way, a hospital that staffs only the average number of beds will quickly realize similar shortages. With our model, hospital management can indicate their risk aversion for having a bed shortage, and from the model output, we can determine the number of beds required to achieve it.

The data used for the model were extracted from the hospital’s patient management system for the calendar year 2008. The model calculations were facilitated with Microsoft Visual Basic.

Inherent to the model are a number of assumptions that are discussed to provide perspective on what the model can and cannot be used for. Patients are counted as “occupying a bed” on the day of admission, not on the day of discharge; thus, issues with overlapping patients on the same day are not quantified. Emergency patients are not explicitly modeled; however, using historical data, their impact is added to model output. Furthermore, because this project is completed at a specialty cancer hospital, the presence of emergency patients is low. It is assumed that the MSS does not change from week to week. In reality, the MSS does occasionally change to reflect seasonal patterns. During these periods, certain OR blocks are canceled, meaning our model overestimates the ward occupancy during these weeks, but nevertheless still may be helpful as an upper bound.

Finally, the model assumes that there is always a bed available for a patient after surgery. This implies that procedures are never canceled because of bed shortages.

This is perhaps an idealized situation, although it is representative of hospitals that call in additional staff when demand is higher than expected (as is the practice of the hospital in this case study). The frequency of “calling in additional staff” follows from the model. Because, in practice, additional staff may not always be available (and procedures are occasionally canceled as a result), the model output may overestimate the bed requirements, although only slightly when cancellations are uncommon.

RESULTS

This section is divided into 2 subsections. The first subsection discusses ward occupancy projected by the model during the MSS development process. We show ward occupancy projections from the original MSS proposal and from the MSS proposal that staff chose to implement (which we refer to as the implemented MSS). In the second subsection, we compare the ward occupancy projected by the model for the implemented MSS with the ward occupancy observed after it was implemented.

Projected Results

NKI-AVL has 2 wards for treating surgical patients, Ward A and Ward B, with a combined physical capacity of 100 beds. Management strives to staff enough beds such that for 90% of the weekdays there is sufficient coverage. In other words, they staff for the 90th percentile of demand and thereby assert that their accepted risk for needing to call in additional staff is 10%. Figure 5 illustrates the 90th percentile demand for staffed beds on each of the wards, resulting from the original MSS proposal. As is clear from the figure, the staffing requirements are relatively balanced across weekdays for Ward B. This is not the case for Ward A. On Ward A, the occupancy is relatively low on Monday and Tuesday, and relatively high on Thursday, Friday, and Saturday.

This projected demand for staffed beds concerned the ward manager because such an unbalanced demand profile makes staff scheduling and ward operations difficult. Early in the week, beds would be underutilized, whereas later in the week, beds would become highly utilized and the risk of a shortage would increase. Such peaks and valleys represent variation in the system that possibly could be eliminated with a different MSS. This variation leads to...
significant problems, particularly as the wards approach peak capacity. For example, when inpatient wards reach their peak capacity and a patient admission is pending, staff often scramble to try to make a bed available. If one cannot be made available, additional staff are called in (or in rare cases when additional staff cannot be found, the elective surgery is canceled), which causes extra work for OR planners, wasted time for surgeons, and extra anxiety for patients. When a bed was made available, it often means a patient was transferred from one ward to another (often to a ward capable of caring for the patient but not the designated one) or discharged. Either way, extra work is required by ward staff and there is a disruption in patient care. Although completely eliminating such problems is likely not possible without an exorbitant amount of resources (because of the variance), sound planning ahead of time may help to minimize occurrences.

Upon discussing the model output, all project team members agreed that the original MSS, although appropriate for the OR, was not ideal for the wards. The discussion then moved to how to correct the imbalance across the weekdays by changing the assignment of OR blocks to specialties. Modifications to the original MSS were made by considering what changes were possible within the restrictions of the OR.

Eventually, after considering several MSS proposals, the process led to an MSS (the implemented MSS) that was agreeable to all staff members. The implemented MSS fit within the restrictions of the OR (e.g., physician schedules and equipment availability) and, as illustrated in Figure 6, resulted in a more balanced ward occupancy. Comparing the implemented MSS with the original MSS, the implemented MSS dampened the fluctuation on Ward A by decreasing occupancy on Thursday, Friday, and Saturday, and increasing it on Monday and Tuesday. With the implemented MSS, the model predicted that no days would require >47 staffed beds, which reduced the maximum from 49 (predicted for the original MSS). Furthermore, the implemented MSS ensured that the staffing requirements remained relatively balanced across the working days for both wards.

**Observed Results**

The ward occupancy was observed over a 33-week period after OR 6 was fully operational and when the implemented MSS was in use. From these data, probability distributions of beds used for each day of the MSS cycle were derived. Using $\chi^2$ goodness-of-fit tests, these observed distributions were compared with those projected by the model. For Ward B, 6 of the 7 projected distributions (one for each day of the MSS cycle) were found to be a good fit for the observed data at a level $\alpha = 0.05$, whereas for the seventh day, this was true at a level $\alpha = 0.2$. For Ward A, the tests found the projected distributions to be a good fit at levels $\alpha = 0.15$ (for 3 days), 0.25 (for 2 days), and 0.35 (for 2 days). At these $\alpha$ levels, we conclude that the observed ward occupancy is well predicted by the model. Explanations for the poorer fit of Ward A data are discussed in the following paragraphs where the 90th percentiles (desired staffing level) are compared for the observed and projected results.

Figures 7 and 8 compare the projected ward occupancy with the observed ward occupancy during the 33-week period. Figure 7 displays results for Ward A and Figure 8 for Ward B.

As is observable in Figures 7 and 8, the data indicate that both wards are indeed experiencing a balanced ward occupancy across the weekdays. However, it is also observable that our model overestimated the number of beds required, in Ward A by approximately 16%. The overestimate is attributable to an unexpected increase in short-stay patients during the period of measurement. Had this change in patient mix been expected at the time the projections were made (and model input altered to reflect it), the overestimate would not have been observed. This highlights how process variables may change over time and the importance of estimating this and reflecting it in the model input.

As a final note on the model results, consider if hospital management decided to staff only for the average number of beds projected to be needed for 6 ORs. In this case, 32 beds would be assigned to Ward A and 29 beds to Ward B. This would have led to a bed shortage on 51% of the days. This illustrates the importance of considering probability distributions in hospital planning.

**DISCUSSION**

With the approach discussed in this article, a new MSS was developed for NKI-AVL that reduced the fluctuations in...
the daily ward census, creating a more balanced workload on the wards. The roll-out of the new MSS corresponded with the opening of an additional OR, which was expected to overwhelm the wards. By using the process described herein to develop an MSS that accounted for the inpatient wards, peaks in ward occupancy were reduced. As such, capacity will be used more efficiently and the hospital is given the means to support the additional OR without a major expansion in the wards.

The main benefit of using the model was to be able to quantify the concerns of ward staff, thereby providing a platform from which they could begin to negotiate a solution. Staff was quick to embrace the model output, particularly after seeing several modifications to the original MSS, at which point they were able to roughly predict model output for a given modification. For example, on Thursdays and Fridays the wards tended to be crowded with patients. To remedy this, specialties that completed many cases per OR block were removed from Thursday and Friday OR blocks and assigned to OR blocks earlier in the week. To accommodate these changes, specialties that complete a relatively small number of cases per OR block were moved to Thursday and Friday. Once staff could foresee the impact of swapping one surgical OR block assignment with another, the implemented MSS came quickly.

The process and the case study illustrate how concepts from operations research and industrial engineering can contribute to hospital operations. The close collaboration between the clinical staff and modelers was of benefit to both groups. The clinical staff was able to directly see how the model results were affected by the model input. This insight helped to build their understanding and interest in the model, which in turn led to its credibility. For the modelers, the day-to-day involvement of clinical staff ensured that a valid model was developed, giving them firsthand knowledge of the actual system. This working relationship ensured model validity and credibility, which contributed a great deal toward implementation.

In this project we treated the equipment and physician schedule restrictions as unchangeable. It is possible that further improvements in the ward occupancy could have been achieved if these restrictions were relaxed. In this way, the model can be used to illustrate the benefits of buying an extra piece of equipment or of changing physicians’ schedules. An additional restriction, which if relaxed may have allowed further improvements, is the assignment of wards to surgical specialties. In other words, in addition to changing when a specialty operates, it may prove advantageous to change which ward the patients are admitted to after surgery. Finally, we chose the best MSS from those created through swapping OR block and surgical specialty assignments. It is possible that a search heuristic may have found a better MSS, although it would have required the many surgical department restrictions to be modeled and the more complex model may not have garnered the same level of staff understanding and support.

The work in this article, and in particular the model, provides a basis for future redesign of the MSS when conditions change and further adaptations are necessary. It is also readily repeatable in other hospitals and relies only on readily available data. The model is sufficiently robust to accommodate a number of different circumstances that may be found at different hospital settings. Examples of such circumstances and how to cope with them include: larger hospitals by increasing the number of OR blocks, specialties with presurgical lengths of stay by allowing \( n \) (see Appendix) to be a negative number, patients switching wards by allowing \( h_{ij}(x) \) (see Appendix) to be ward specific, and specialties with highly varying patient populations by changing how patient cohorts are defined (see discussion in Methods section).

The method in which MSS modifications are proposed can be different than the manner described in this article. Our approach amounted to trial and error, which was appropriate given that there are only 6 ORs and a limited number of possible MSS proposals due to the restrictions of the OR. Other hospitals, particularly larger hospitals, may prefer a more structured method such as a search heuristic as discussed above.

Finally, variants of this model have been applied in 3 other Dutch hospitals. At 2 of the hospitals, the model was used to improve the workload balance on the inpatient wards. In the third hospital, it was used to evaluate the allocation of beds to wards. Furthermore, the possibility of implementing the model as part of the hospital’s business intelligence software to allow management to independently evaluate changes to the MSS based on the most recent data is explored.

**APPENDIX**

The formal model described in this Appendix is explained in greater detail in the article by Vanberkel et al.\(^{18}\) Let the master surgical schedule (MSS) be defined such that \( b_{ij}(t) \) is an operating room (OR) block where \( i = 1,2, \ldots, I \) indexes the ORs and \( q = 1,2, \ldots, Q \) the days in a cycle. Let each surgical specialty \( j \) be characterized by 2 parameters \( c_j \) and \( d_{jn} \), where \( c_j \) is a discrete distribution for the number of procedures performed in 1 OR block and \( d_{jn} \) the probability that a patient, who is still in the ward on day \( n \), is to be discharged that day \( (n = 0,1, \ldots, L_j) \) where \( L_j \) denotes the maximum length of stay for specialty \( j \).

Using \( c_j \) and \( d_{jn} \) as model inputs, for a given MSS, the probability distribution for the number of recovering patients on each day \( q \) can be computed. The required number of beds is computed with the following 3 steps. Step 1 computes the distribution of recovering patients from a single OR block of a specialty \( j \); i.e., we essentially precalculate the distribution of recovering patients expected from an OR block of a specialty. In step 2, we consider a given MSS and use the result from step 1 to compute the distribution of recovering patients given a single cycle of the MSS. Finally, in step 3, we incorporate recurring MSSs and compute the probability distribution of recovering patients on each day \( q \).

**Step 1**

For each specialty \( j \) we use the binomial distribution to compute the number of beds required from the day of surgery \( n = 1 \) until \( n = L_j \). Because we know the probability distribution for the number of patients having surgery \( (c_j) \),...
which equates to the number of beds needed on day $n = 0$, we can use the binomial distribution to iteratively compute the probability of needing beds on all days $n > 0$. Formally, the distribution for the number of recovering patients on day $n$ is recursively computed by:

$$h_m^n(x) = \begin{cases} c^n(x) & \text{when } n = 0 \\ \sum_{k=0}^{\infty} \binom{x}{k} \left( d_{n-1}^i \right)^{x-k} \left( 1 - d_{n-1}^i \right)^k h_{n-1}^i(k) & \text{otherwise.} \end{cases}$$

**Step 2**

We calculate for each OR block $b_{iq}$ the impact this OR block has on the number of recovering patients in the hospital on days $q, q + 1, \ldots$ If $j$ denotes the specialty assigned to OR block $b_{ij}$, then let $h_m^j$ be the distribution for the number of recovering patients of OR block $b_{ij}$ on day $m = 1, 2, \ldots, Q_i$, $Q_i + 1, Q_i + 2, \ldots$. It follows that,

$$h_m^j(x) = \begin{cases} h_{m-q}^j(x) & \text{if } q \leq m < l + q \\ 0 & \text{otherwise} \end{cases}$$

where 0 means $h_m^j(0) = 1$ and all other probabilities $h_m^j(l) = 0$, $l > 0$.

Let $H_m^q$ be a discrete distribution for the total number of recovering patients on day $m$ resulting from a single MSS cycle. Because recovering patients do not interfere with each other, we can simply iteratively add the distributions of all the OR blocks corresponding to the day $m$ to get $H_m^q$. Adding 2 independent discrete distributions is done by discrete convolutions, which we indicated by $*$. For example, let $A$ and $B$ be 2 independent discrete distributions. Then $C = A*B$, which is computed by:

$$C(x) = \sum_{k=0}^{\infty} A(k) B(x - k)$$

where $\tau$ is equal to the largest $x$ value with a positive probability that can result from $A*B$ (e.g., if the maximum value of $A$ is 3 and the maximum value of $B$ is 4, then when convoluted, the maximum value of the resulting distribution is $7$; therefore, in this example $\tau = 7$). Using this notation, $H_m$ is computed by:

$$H_m^q(x) = h_m^1,1 * h_m^2,2 * \ldots * h_m^q,0 * h_m^3,1 * \ldots * h_m^Q,0$$

**Step 3**

We now consider a series of MSSs to compute the steady-state probability distribution of recovering patients. The cyclic structure of the MSS implies that patients receiving surgery during one cycle may overlap with patients from the next cycle. In the case of a small $Q_i$ patients from many different cycles may overlap.

In step 2, we have computed $H_m^q$ for a single MSS in isolation. Let $M$ be the last day in which there is still a positive probability that a recovering patient is present in $H_m^q$. To calculate the overall distribution of recovering patients when the MSS is repeatedly executed, we must consider $\frac{M}{Q_i}$ consecutive MSSs. Let $H_{SS}^q$ denote the probability distribution of recovering patients on day $q$ of the MSS cycle, resulting from the consecutive MSSs. Because the MSS does not change from cycle to cycle, $H_{SS}^q$ is the same for all MSS cycles. Such a result, where the probabilities of various states remain constant over time, is referred to as a steady-state result. Using discrete convolutions, $H_{SS}^q$ is computed by:

$$H_{SS}^q(x) = H_q^* H_{q+1}^* H_{q+2}^* \ldots * H_{q+M^Q}^* \ldots$$

To determine the demand for ward beds from the variable $H_{SS}^q$, consider the following example. Let the staffing policy of the hospital be such that they staff for the 90th percentile of demand and let $D_q$ denote the 90th percentile of demand on day $q$. It follows that $D_q$ is also the number of staffed beds needed on day $q$, and is computed from $H_{SS}^q$ as follows:

$$D_q = \max \{ x | H_{SS}^q(x) \leq 0.9 \}$$

**REFERENCES**


**DISCLOSURES**

Name: Peter T. Vanberkel, MASc.

Contribution: This author helped design the study, conduct the study, analyze the data, and write the manuscript.

Attestation: Peter T. Vanberkel has seen the original study data, reviewed the analysis of the data, approved the final manuscript, and is the author responsible for archiving the study files.

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