

# Spectral Representations of Fingerprint Minutiae Subsets

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**Abstract**—The investigation of the privacy protection of biometric templates gains more and more attention. The spectral minutiae representation is a novel method to represent a minutiae set as a fixed-length feature vector, which is invariant to translation, and in which rotation and scaling become translations, so that they can be easily compensated for. These characteristics enable the combination of fingerprint recognition systems with template protection schemes that require as an input a fixed-length feature vector. However, the limited overlap of a fingerprint pair can reduce the performance of the spectral minutiae representation algorithm. Therefore, in this paper, we introduce the spectral representations of fingerprint minutiae subsets to cope with the limited overlap problem. In the experiment, we improve the recognition performance from 0.32% to 0.12% in equal error rate after applying the spectral representations of minutiae subsets algorithm.

## I. INTRODUCTION

Biometrics technologies are developing rapidly in order to meet high security requirements. Among various biometric identifiers, such as face, signature and voice, the fingerprint has higher levels of distinctiveness and performance [1] and it is the most commonly used biometric modality. Many fingerprint recognition systems are based on the use of a minutiae set [2], [3]. Minutiae are the endpoints and bifurcations of fingerprint ridges. They are known to remain unchanged over an individual's lifetime and allow a very discriminative classification of fingerprints.

In the recent years, the privacy protection of biometric templates has drawn more and more attention of researchers [4], [5]. To enable the combination of fingerprint recognition systems with recently developed template protection schemes based on fuzzy commitment and helper data schemes, such as [6] and [7], a fixed-length feature vector representation of a biometric modality is required as an input<sup>1</sup>. The spectral minutiae [9] represents a minutiae set as a fixed-length feature vector, which is invariant to translation, and in which rotation and scaling become translations. These characteristics enable the combination of fingerprint recognition systems with template protection schemes and allow for faster matching as well.

However, in template protection systems where encrypted templates are stored, it is not possible to align the *reference* and *test* fingerprints using minutiae information in the spectral

minutiae fingerprint recognition system. The study presented in [9] shows that the recognition errors occur when the percentage of corresponding minutiae is below 75 in an *ideal* situation (that is, no other errors present such as spurious and missing minutiae, minutiae location errors). Therefore, the limited overlap between a fingerprint pair can degrade the recognition performance of the spectral minutiae fingerprint recognition system. Some fingerprint recognition algorithms use reference points (such as core, delta) to pre-align fingerprints [6], [8]. However, these methods have problems to cope with. First, some fingerprints do not have such reference points. Second, the reference points may not appear in the fingerprint images during acquisition. Third, the reference points detector may fail to locate the points.

Therefore, in this paper, we present a *spectral representation of minutiae subset* algorithm to cope with the limited overlap problem, that does not rely on reference points. In this method, we generate several subsets from *one* minutiae set and then apply the spectral minutiae representation to the subsets. The corresponding minutiae percentage between the minutiae subsets can increase. In this way, by applying the spectral representations of minutiae subsets, our system is more robust against the limited overlap problem.

This paper is organized as follows. First, a review of the spectral minutiae representation is presented in Section II. Next, in Section III, the spectral representations of minutiae subsets algorithm is introduced. Finally, Section IV presents the experimental results and we draw conclusions in Section V.

## II. BACKGROUND

The spectral minutiae representation is based on the shift, scale and rotation properties of the two-dimensional continuous Fourier transform. In [9], the concept of and algorithms for two representation methods are introduced: the *location-based spectral minutiae representation* (SML) and the *orientation-based spectral minutiae representation* (SMO).

### A. Spectral Minutiae Representations

Assume we have a fingerprint with  $Z$  minutiae. In SML, with every minutia, a function  $m_i(x, y) = \delta(x - x_i, y - y_i)$ ,  $i = 1, \dots, Z$  is associated where  $(x_i, y_i)$  represents the location of the  $i$ -th minutia in the fingerprint image. Thus, in the spatial domain, every minutia is represented by a Dirac pulse. The Fourier transform of  $m_i(x, y)$  is given by:

<sup>1</sup>Other template protection systems exist [8] that do not pose this fixed-length feature vector requirement.

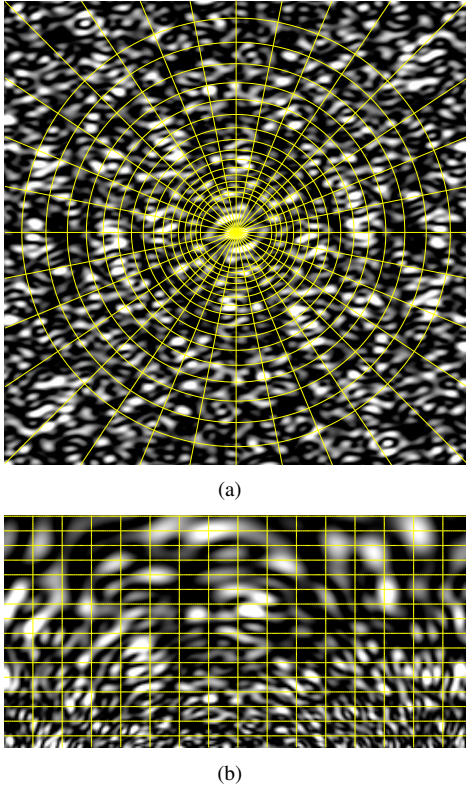


Fig. 1. Illustration of the polar-logarithmic sampling (SML spectra). (a) the Fourier spectrum in a Cartesian coordinate and a polar-logarithmic sampling grid; (b) the Fourier spectrum sampled on a polar-logarithmic grid.

$$\mathcal{F}\{m_i(x, y)\} = \exp(-j(\omega_x x_i + \omega_y y_i)), \quad (1)$$

and the location-based spectral minutiae representation is defined as

$$\mathcal{M}_L(\omega_x, \omega_y) = \sum_{i=1}^Z \exp(-j(\omega_x x_i + \omega_y y_i)). \quad (2)$$

In order to reduce the sensitivity to small variations in minutiae locations in the spatial domain, we use a Gaussian low-pass filter to attenuate the higher frequencies. This multiplication in the frequency domain corresponds to a convolution in the spatial domain where every minutiae is now represented by a Gaussian pulse.

Following the shift property of the Fourier transform, the magnitude of  $\mathcal{M}$  is taken in order to make the spectrum invariant to translation of the input and we obtain

$$\left| \mathcal{M}_L(\omega_x, \omega_y; \sigma_L^2) \right| = \left| \exp\left(-\frac{\omega_x^2 + \omega_y^2}{2\sigma_L^2}\right) \sum_{i=1}^Z \exp(-j(\omega_x x_i + \omega_y y_i)) \right|. \quad (3)$$

The location-based spectral minutiae representation (SML) only uses the minutiae location information. However, including the minutiae orientation as well may give better

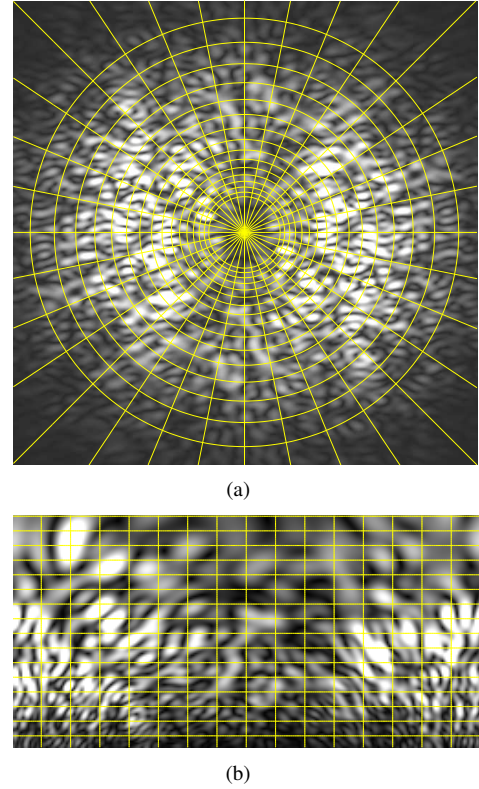


Fig. 2. Illustration of the polar-logarithmic sampling (SMO spectra). (a) the Fourier spectrum in a Cartesian coordinate and a polar-logarithmic sampling grid; (b) the Fourier spectrum sampled on a polar-logarithmic grid.

discrimination. Therefore, it can be beneficial to also include the orientation information in our spectral representation. The orientation  $\theta$  of a minutiae can be incorporated by using the spatial derivative of  $m(x, y)$  in the direction of the minutiae orientation. Thus, to every minutiae in a fingerprint, a function  $m_i(x, y, \theta)$  is assigned being the derivative of  $m_i(x, y)$  in the direction  $\theta_i$ , such that

$$\mathcal{F}\{m_i(x, y, \theta)\} = j(\omega_x \cos \theta_i + \omega_y \sin \theta_i) \cdot \exp(-j(\omega_x x_i + \omega_y y_i)). \quad (4)$$

As in the SML algorithm, using a Gaussian filter and taking the magnitude of the spectrum yields

$$\left| \mathcal{M}_O(\omega_x, \omega_y; \sigma_O^2) \right| = \left| \exp\left(-\frac{\omega_x^2 + \omega_y^2}{2\sigma_O^2}\right) \sum_{i=1}^Z j(\omega_x \cos \theta_i + \omega_y \sin \theta_i) \cdot \exp(-j(\omega_x x_i + \omega_y y_i)) \right|. \quad (5)$$

In order to obtain the final spectral representations, the continuous spectra (3) and (5) are sampled on a polar-logarithmic grid. In the radial direction  $\lambda$ , we use  $M = 128$  samples between  $\lambda_1 = 0.1$  and  $\lambda_h = 0.6$ . In the angular direction  $\beta$ , we use  $N = 256$  samples uniformly distributed between  $\beta = 0$  and  $\beta = \pi$ . Because of the symmetry of the Fourier transform

for real-valued functions, using the interval between 0 and  $\pi$  is sufficient. This polar-logarithmic sampling process is illustrated in Figures 1 and 2. For each spectrum, the horizontal axis represents the rotation angle of the spectral magnitude (from 0 to  $\pi$ ); the vertical axis represents the frequency of the spectral magnitude (the frequency increases from top to bottom). The resulting representation in the polar-logarithmic domain is invariant to translation, while rotation and scaling of the input have become translations along the polar-logarithmic coordinates.

### B. Spectral Minutiae Matching

Let  $R(m, n)$  and  $T(m, n)$  be the two sampled minutiae spectra in the polar-logarithmic domain respectively achieved from the *reference* fingerprint and *test* fingerprint. Both  $R(m, n)$  and  $T(m, n)$  are normalized to have zero mean and unit energy. We use the two-dimensional correlation coefficient between  $R$  and  $T$  as a measure of their similarity.

In practice, the input fingerprint images are rotated and might be scaled (for example, depending on the sensor that is used to acquire an image). Since the minutiae spectra are translation invariant, but not rotation and scaling invariant, this method has to test a few different combinations of rotation and scaling, which are translations in the minutiae spectra. To be specific, the scaling becomes the shift (or translation) in the vertical direction, and the rotation becomes the circular shift in the horizontal direction. We denote  $T(m-i, n-j)$  as a shifted version of  $T(m, n)$ , with a shift of  $i$  in the vertical direction and a circular shift  $j$  in the horizontal direction. Then, the correlation coefficient between  $R$  and  $T$  is defined as:

$$C^{(R,T)}(i, j) = \frac{1}{MN} \sum_{m,n} R(m, n)T(m-i, n-j). \quad (6)$$

In most fingerprint databases, there is no scaling difference between the fingerprints, or the scaling can be compensated for on the level of the minutiae sets [10]. Therefore, in practice only a few rotations need to be tested. We use the fast rotation shift searching algorithm that was presented in [11] (which tests 9 rotation possibilities in a range of  $-10^\circ$  to  $+10^\circ$ ) and finally the maximum score from the different combinations is the final matching score between  $R$  and  $T$ ,

$$S^{(R,T)} = \max_j \{C^{(R,T)}(0, j)\}, \quad -15 \leq j \leq 15. \quad (7)$$

## III. SPECTRAL REPRESENTATIONS OF MINUTIAE SUBSETS

### A. Fingerprint Minutiae Subsets Generation

In this method, we generate several minutiae subsets from one minutiae set by selecting minutiae in a number of rectangular areas of the same size.

Assume we have a minutiae set  $M_{\text{all}}$  with  $Z$  minutiae,  $\{(x_i, y_i)\}, i = 1, \dots, Z$ , with  $(x_i, y_i)$  the location of the  $i$ -th minutia. Let  $x_{\min}, x_{\max}, y_{\min}, y_{\max}$  denote the boundaries of the minutiae locations and  $d_x, d_y$  the sides length of the fixed size rectangular area. Then, we can generate four minutiae subsets  $M_{\text{ul}}, M_{\text{ur}}, M_{\text{bl}}$  and  $M_{\text{br}}$  at the upper-left

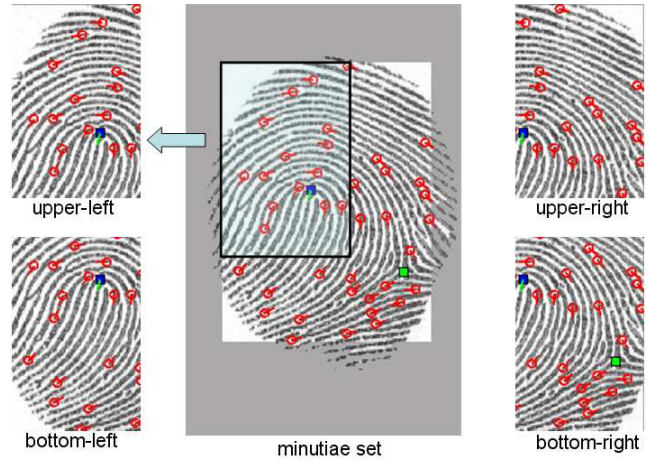


Fig. 3. Illustration of one minutiae set and its four subsets.

(ul), upper-right (ur), bottom-left (bl), bottom-right (br) part of the minutiae set, as

$$\begin{aligned} M_{\text{ul}} &= \{(x, y) | x_{\min} \leq x \leq x_{\min} + d_x, y_{\min} \leq y \leq y_{\min} + d_y\}, \\ M_{\text{ur}} &= \{(x, y) | x_{\max} - d_x \leq x \leq x_{\max}, y_{\min} \leq y \leq y_{\min} + d_y\}, \\ M_{\text{bl}} &= \{(x, y) | x_{\min} \leq x \leq x_{\min} + d_x, y_{\max} - d_y \leq y \leq y_{\max}\}, \\ M_{\text{br}} &= \{(x, y) | x_{\max} - d_x \leq x \leq x_{\max}, y_{\max} - d_y \leq y \leq y_{\max}\}. \end{aligned}$$

This procedure is illustrated in Figure 3.

### B. Matching Procedure

During the enrollment, the spectral minutiae representations of the *reference* minutiae set  $M_{\text{ref,all}}$  and its subsets  $M_{\text{ref,ul}}, M_{\text{ref,ur}}, M_{\text{ref,bl}}, M_{\text{ref,br}}$  are stored as templates, denoted as  $R_{\text{all}}, R_{\text{ul}}, R_{\text{ur}}, R_{\text{bl}}, R_{\text{br}}$ , respectively. During the verification, we use the following procedure to verify the *test* fingerprint.

1. The spectral minutiae representations of the test minutiae set  $M_{\text{test,all}}$  and its subsets  $M_{\text{test,ul}}, M_{\text{test,ur}}, M_{\text{test,bl}}, M_{\text{test,br}}$  are taken respectively, denoted as  $T_{\text{all}}, T_{\text{ul}}, T_{\text{ur}}, T_{\text{bl}}$  and  $T_{\text{br}}$ .
2. Calculate the matching score  $S_{\text{all}}$  between  $R_{\text{all}}$  and  $T_{\text{all}}$  following Equation (7),  $S_{\text{all}} = S^{(R_{\text{all}}, T_{\text{all}})}$ .
3. Calculate 16 matching scores between minutiae subsets and take the largest score as the spectral minutiae subsets score  $S_{\text{sub}}$ , that is,

$$S_{\text{sub}} = \max_{R, T} \{S^{(R,T)} | R \in \{R_{\text{ul}}, R_{\text{ur}}, R_{\text{bl}}, R_{\text{br}}\}, T \in \{T_{\text{ul}}, T_{\text{ur}}, T_{\text{bl}}, T_{\text{br}}\}\}. \quad (8)$$

4. Implement a score level sum-rule fusion of  $S_{\text{all}}$  and  $S_{\text{sub}}$ .
5. The steps 1-4 are applied to SML and SMO respectively, and finally, a score level sum-rule fusion of the SML and SMO results is applied to achieve the final matching score.

## IV. EXPERIMENTS

We test the spectral representations of fingerprint minutiae subsets in a verification setting. The matching performance of a fingerprint verification system can be evaluated by the *false acceptance rate* (FAR), the *false rejection rate* (FRR), and

TABLE I  
EXPERIMENTAL SETTINGS.

SML	$\sigma_L$	0
	$\lambda_l$	0.1
	$\lambda_h$	0.6
SMO	$\sigma_O$	4.24
	$\lambda_l$	0.01
	$\lambda_h$	0.56
	$d_x$	120 (pixel)
	$d_y$	180 (pixel)

TABLE II  
RESULTS ON THE MCYT DATABASE.

Methods	EER	GAR	
		FAR = 0.1%	FAR = 0%
No Minutiae Subsets	0.32%	99.5%	99.1%
Minutiae Subsets	0.12%	99.9%	99.7%

the *equal error rate* (EER). When the decision threshold of a biometric security system is set such that the FAR and FRR are equal, the common value of FAR and FRR is referred to as the EER. In this paper, we use FAR, EER and the *genuine accept rate* (GAR),  $GAR = 1 - FRR$ , as performance indicators of our scheme.

The proposed algorithms have been evaluated on the MCYT [12] fingerprint database. The fingerprint data that we used from MCYT are obtained from 100 individuals (person ID from 0000 to 0099 in MCYT, and finger ID for each individual is 0) and each individual contributed 12 samples. The minutiae sets were obtained by the VeriFinger minutiae extractor [3]<sup>2</sup>. During the test, for each comparison, we chose two fingerprints from the data set: one as a *reference* fingerprint, another one as a *test* fingerprint. For matching genuine pairs, we used all the possible combinations. For matching imposter pairs, we chose the first sample from each identity. In total, we implement 6600 genuine comparisons and 4950 imposter comparisons. The experimental settings are shown in Table I. The final results are shown in Table II and the ROC curves are shown in Figure 4. For comparison, the result of the spectral minutiae representation without using minutiae subsets is also shown.

From the results, we can see that the recognition performance of the spectral minutiae representation improves after applying the spectral representations of the minutiae subsets. This shows that by generating the minutiae subsets, the corresponding minutiae percentage between the minutiae subsets increase compared with the one between the total minutiae sets, and this results in an improved recognition accuracy. By applying the spectral representations of minutiae subsets, our system is more robust against the limited overlap problem.

<sup>2</sup>VeriFinger Extractor Version 5.0.2.0 is used.

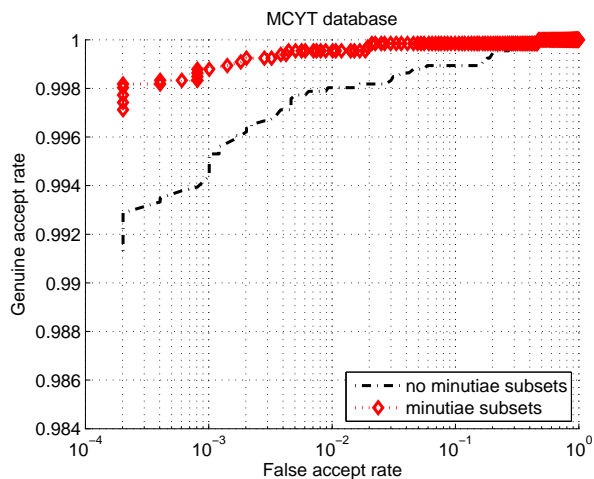


Fig. 4. ROC curves.

## V. CONCLUSIONS

In fingerprint recognition systems, a limited overlap between the reference and test fingerprints is unavoidable. To make the spectral minutiae representation system more robust against the limited overlap problem, we introduce the algorithm of the spectral representations of fingerprint minutiae subsets. The experimental result shows a promising enhancement in recognition accuracy.

The algorithm we present in this paper does not rely reference points. Therefore, this algorithm does not suffer from the problems that can be caused by reference points, such as the failure of reference points detection. Moreover, this method can be easily integrated to the large number of existing minutiae databases without requiring additional fingerprint image based information.

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