Abstract

This paper describes the various design aspects for micromechanical sensors consisting of a structure with encapsulated built-in resonant strain gauges. Analytical models are used to investigate the effect of device parameters on the behaviour of a pressure sensor and a force sensor. The analyses indicate that the sealing cap can have a strong degrading effect on the device performance if the thicknesses of the cap and of the supporting structure are of the same order of magnitude. A novel design, employing bossed structures, is described, which reduces the design complexity and virtually eliminates the influence of the cap on the sensitivity of the sensor.

Introduction

In a mechanical sensor using resonant strain gauges, an external load such as pressure, force or acceleration is converted into a shift of the resonance frequency of the gauge. Resonant strain gauges are used as the strain-sensing elements of the sensor in a similar way as piezoresistive strain gauges are used in more conventional devices. Resonant sensors provide a frequency-shift output and are becoming increasingly attractive in the precision measurement field [1]. They can offer excellent stability, resolution and accuracy. Further, a frequency output provides easy interfacing with digital systems, and immunity to fluctuations of the intensity of the signal. To be effectively employed as a measurement device, the structure should only respond to changes in the load to be measured. Interaction of a load with the resonator can be done through a perturbation of its kinetic or potential energy [2]. For the class of sensors considered in this paper, a shift in the resonance frequency of the strain gauge is caused by potential energy perturbations. This puts strong demands on the design of the structure to eliminate kinetic energy perturbations such as density changes of the surrounding medium.

One of the first resonant strain gauge type pressure transducers was described by Belyaev et al. in 1965 [3]. In their design, the strain gauge is mounted on top of a diaphragm by means of two brackets. Similar designs were reported by Greenwood in 1984 [4] and by Thornton et al. in 1988 [5]. Silicon bulk micromachining was used to fabricate these devices. A novel design of a silicon pressure sensor was reported by Ikeda et al. in 1988 [6]. Here, the resonators are held in evacuated microcavities on the surface of the diaphragm to isolate them from the surroundings. Single-crystal silicon is used as the construction material and selective epitaxial growth techniques combined with high boron etch stops are used to fabricate the device. An alternative way of fabricating sealed resonators was reported by Guckel et al. in 1989 [7]. They use sacrificial layer etching and reactive sealing techniques with LPCVD fine-grained polysilicon as a construction material. Both technologies are very attractive for batch fabrication of the devices.
This paper deals with design considerations of mechanical sensors using built-in encapsulated resonant strain gauges. The characteristic behaviour of the strain gauges itself is discussed. Moreover, the influence of the sealing cap on the mechanical behaviour of the device is modelled for several example structures including a novel design employing a bossed structure. Material aspects, fabrication technologies and means of excitation and detection of the vibration are not a subject of this paper.

Resonant Strain Gauges

A resonant strain gauge consists of a mechanical resonator mounted on a supporting structure, e.g., a diaphragm, at the (two) ends of the gauge. Axial elongation is induced in the gauge by a deformation of the supporting structure, resulting in a shift of the resonance frequency of the gauge. The gauge or the resonator can be a doubly supported beam, a double-ended tuning fork, a triple-beam structure or an H-shaped resonator, all operating in a bending mode vibration [1-7]. A resonant strain gauge can be characterized by its gauge factor $G$ and its mechanical quality factor $Q$.

The gauge factor is a measure of the sensitivity of the gauge, defined by

$$G = \left. \frac{1}{f} \frac{df}{d\varepsilon} \right|_{\varepsilon = \varepsilon_0}$$  \hspace{1cm} (1)

where $f$ is the resonance frequency, $\varepsilon$ the strain, and $\varepsilon_0$ the residual strain, i.e., the strain level at the operating point.

To gain insight into the frequency dependence of a strain gauge on design parameters, a prismatic (wide) beam with a rectangular cross section, rigidly clamped at both ends, is taken as an example. The resonance frequency $f$ of the beam with applied strain $\varepsilon$ can be expressed as

$$f \approx 1.028 \left[ \frac{E}{\rho (1 - v^2)} \right]^{1/2} \left[ \frac{h}{l^2} \right] \times \left[ 1 + 0.295 \varepsilon (1 - v^2) \left( \frac{l}{h} \right)^2 \right]^{1/2} \hspace{1cm} (2)$$

where $E$, $\rho$ and $v$ are Young’s modulus, specific mass and Poisson’s ratio of the beam material, and $h$ and $l$ are the thickness and length of the beam, respectively. Equation (2) is an approximation derived from Rayleigh’s quotient, substituting the first mode shape for zero applied axial load as the approximate beam deflection shape. An expression for the gauge factor $G$ can now easily be found

$$G = 0.5 \frac{0.295(1 - v^2) \left( \frac{l}{h} \right)^2 \varepsilon}{1 + 0.295 \varepsilon_0 (1 - v^2) \left( \frac{l}{h} \right)^2} \hspace{1cm} (3)$$

For highly sensitive devices, a high gauge factor is desirable. It is obvious from eqn (3) that this is accomplished for a high aspect ratio $(l/h)$ and/or low residual strain levels $\varepsilon_0$ of the gauge. This is also illustrated in Fig 1.

Equation (2) gives the frequency of the gauge in a vacuum. If the gauge is immersed in a gas or liquid, its resonance frequency will be lower due to an increase of the effective mass [9]. Further, as a result of the increased damping, the $Q$ factor will be reduced. Especially if the flexurally vibrating resonator is close to another stationary surface, e.g., if sacrificial layer etching techniques are used for fabrication, high energy losses caused by squeeze-film damping result [10, 11]. For a good sensor performance, a high $Q$ is desired, which means a good frequency resolution, an

![Fig 1](image_url)
excellent rejection of external mechanical noise and a minimized dependence of the sensor performance on the characteristics of the electronic circuitry used to sustain the oscillation. Hence, the resonator needs to be housed in a hermetically sealed evacuated cavity to eliminate the disturbing influences of the surrounding medium on the gauge behaviour and to achieve an optimum sensor performance.

The amplitude of the vibration at resonance is another relevant issue. On the one hand, the amplitude needs to be large enough for the vibration to be detected. On the other hand, an increase of the amplitude results in a shift towards higher resonance frequencies due to the 'hard spring effect' [12, 13]. The amplitude is generally determined by the driving power, the mode shape, the efficiency of excitation/detection and the $Q$ factor. A stable amplitude requires a constant $Q$ factor and an AGC amplifier to sustain the oscillation for a given mode shape and excitation/detection scheme.

### Placement of the Resonant Strain Gauges

The resonant strain gauges are subject to axial elongations if the supporting structure deforms under an applied load. In general, the induced axial strain $\varepsilon$ can be expressed as

$$\varepsilon = \varepsilon_N + \frac{z}{l} [\phi_1 - \phi_2]$$

(4)

where $\varepsilon_N$ is a normal strain due to in-plane loads acting on the supporting structure, $z$ is the distance between the mid-plane of the gauge and the neutral plane (as defined for the case of pure bending) of the supporting structure, $l$ is the gauge length and $\phi_1$ and $\phi_2$ are the angles of rotation at the end points of the gauge. The gauge length is generally a given parameter determined by the desired gauge factor (see eqn (3)) and the base frequency (see eqn (2)). The second strain term, i.e., the bending strain, in eqn (4) arises as a result of bending moments acting on the supporting structure. The in-plane loads, responsible for the normal strains, are caused by non-linear, i.e., large deflection, deformations of the supporting structure, or by the sealing cap in the case of improper design of the structure (see below). In general, it is the objective to maximize the bending strains (and at the same time minimize the normal strains). High bending strains can be obtained by mounting the gauge on pillars to increase $z$, e.g., see Thornton et al [5]. Further, the difference in the angles of rotation of the end points can be optimized by proper placement of the gauge. For instance, the gauges should not extend over an inflection point, i.e., a point of maximum (or minimum) angle of rotation, since this results in a reduction of induced bending strain due to strain-averaging effects. In the worst situation, the angles of rotation will cancel and the resulting bending strain will be zero. For a circular diaphragm, clamped at the edge and subjected to a transverse load, the edge of the diaphragm is the optimum place for the gauges since here the curvature is maximum and thus the angle of rotation changes strongly with distance. This also applies for diaphragms with in-plane residual tensile strain.

### Encapsulation

Vacuum encapsulation of the resonant strain gauges can be done in several ways. In the case of a diaphragm-type sensor, the side of the diaphragm with the strain gauges can be sealed entirely, e.g., see Greenwood [4]. However, this approach limits the applications of the structure, e.g., a differential pressure measurement would require two separate sensors. Further, in the case of beam-like structures such as a cantilever beam force sensor or accelerometer, complete sealing is impossible. Local sealing of the resonant strain gauge is a more versatile approach [6, 14]. In this case, the cavity is formed by the supporting diaphragm or beam on the bottom side and a sealing cap on the top side, see Fig. 2. The gauge is embedded in the
supporting structure. Unlike the structure of Greenwood, the cap now forms an integral part of the mechanical structure and always lowers the sensitivity of the sensor. This will be discussed in more detail in the following Sections. It is obvious that the influence of the cap on the sensor behaviour can be ignored if its stiffness is much smaller than the stiffness of the supporting structure. Increasing the stiffness of the supporting diaphragm or beam, however, results in a loss of sensitivity of the sensor. Also, the stiffness of the cap is bounded by a lower limit, since it has to withstand the ambient pressure, e.g., one atmosphere. For instance, in the case of a cap with dimensions $L_c \times w_c \times h_c$ and with $L_c > 3w_c$, regarded as a plate clamped at all four sides, under the condition that the centre deflection should not exceed one tenth of the thickness $h_c$, the upper limit of the aspect ratio $w_c/h_c$ of the cap is approximately 40 when subjected to an ambient pressure of one atmosphere. This means that for a cap width $w_c = 40 \, \mu m$, a minimum thickness $h_c$ of 1 $\mu m$ is required. This sets the minimum gap distance $h_g$ to 0.1 $\mu m$ to avoid collapse. A practical choice for $h_g$ would be in the range 0.5–1.0 $\mu m$. Note that caps wider than 40 $\mu m$ would require a thickness larger than 1 $\mu m$.

**Bossed Structure**

A number of geometries of bossed structures have been used in piezoresistive and capacitive (low) pressure sensor designs to achieve improved performance in terms of (full-scale) sensitivity, linearity, and degree of symmetry for ‘front and back loading’ [15–17]. Since all these improvements are based on mechanical considerations, it is expected that they also apply for mechanical sensors employing resonant strain gauges. In addition to these advantages, the boss offers several new features. If it is used to co-support the sealing cap as indicated in Fig. 3, the influence of the cap on the mechanical behaviour of the structure can be transformed from a (large) local influence to a (small) global influence. The cap is now merely a flexural structural element, parallel to the supporting element. For instance, the influence of the cap on the positions of the inflection points and of the neutral plane, i.e., the plane of zero total strain, is completely eliminated, irrespective of the shape of the cap. As an additional advantage, it simplifies the mechanical analysis and the design of the structure and makes predictions of the sensor characteristics more reliable. More details are given in the next Section. Finally, the bossed structure is also very well suited to implement a differential resonator design [18]. In this design, the sensitivity to the applied mechanical load is doubled, thereby largely gaining back the loss in sensitivity due to the stiffening effect of the boss.

**Modelling Examples and Discussion**

The primary goal of the modelling examples is to study the influence of the cap
parameters on the device performance. To simplify the analysis, one-dimensional models, i.e., beam structures, are investigated. The results are directly applicable to circular or square structures. The average strain induced in the upper fibres of the beam, underneath the cap, is computed as a function of the design parameters. In the analysis, the influence of the gap underneath the strain gauges on the stiffness of the supporting beam is not taken into account. Further, the computations are based on a linear model and the effect of pre-strain is not included.

The results are derived from an analytical model and were confirmed by a two-dimensional numerical plane stress analysis using the COSMOS M finite-element analysis program from Structural Research and Analysis Corporation (SRAC). The numerical analysis indicated no significant effect of the local stress concentrations at the clamped edges and the cap ends on the mechanical behaviour of the structures.

The one-dimensional equivalent of the structure shown in Fig. 2, loaded by a pressure Δp, is taken as the first example. The pressure-induced strain in the resonator is shown in Fig. 4 as a function of the thickness of the supporting beam for various sets of the other device parameters. Figure 5 shows free body diagrams, indicating the forces and moments acting on the structure. The strain for a uniform thickness beam without a cap (curve V) is compared to the strain of the same beam with a cap (curves I, II, III and IV). A significant loss of sensitivity occurs for cap thicknesses (hₖ) close to the thickness of the support beam (hₛ). The degrading influence of the cap can be reduced by choosing (1) a thinner cap, (2) a smaller Young's modulus Eᵥ for the cap material, (3) a smaller gap height h₂, or (4) a thicker beam. Option (4) is less attractive since it will result in an overall loss of sensitivity. A lower limit for the gap height, option (3), is set by the minimum distance required to avoid collapse of the cavity. Options (1) and (2) are limited by the minimum stiffness of the cap required to withstand the ambient pressure. Also,
without a cap, only bending strains, given by the second term in eqn (4), occur. The total strain \( \varepsilon \) for structures with a cap is equal to the sum of a bending strain \( \varepsilon_b \) and a normal strain \( \varepsilon_N \) (see also eqn (4) and Fig 5). The introduction of a normal strain term can be interpreted as a shift of the position of the neutral axis, \( \varepsilon_c \), the axis of zero total strain, of the cross section. The two strain terms are always of opposite sign. For values of \( h_b \) close to \( h_c \), \( \varepsilon_N \) is the dominating term. An increase in \( h_b \) means a stiffer structure, resulting in a decrease in magnitude of both \( \varepsilon_b \) and \( \varepsilon_N \). The normal strain, however, decreases more rapidly than the bending strain, causing the minimum to occur. It is pointed out that a minimum does not always occur, but depends on the load condition and on the device parameters. Finally, the analysis showed that the sensor behaviour is not the same for front and back loading. If the pressure \( \Delta p \) in Fig 4 is applied to the back side instead of the front side, it is found that the magnitude of the normal strains does not change, while the bending strains are (slightly) higher due to additional bending of the localized area of the support underneath the cap.

Figure 6 illustrates the influence of the cap length \( L_c \). A supporting beam, clamped at both ends and applied as a force transducer serves as the example structure. Two different beam designs are discussed, a non-bossed and a centre-bossed structure (see insert). For the bossed structure, the cap is supported by the boss on one end and by the frame or the substrate on the other end. The force is applied at the centre of the beam and the centre deflection and the average strain induced in the gauges are computed as a function of the cap length. Hereby, the gauge length and the gauge position are fixed. The centre deflection gives an indication about the global or overall behaviour of the structure, while the strain provides more local information, \( \varepsilon_c \), underneath the cap.

Compared to the bossed beam, the behaviour of the non-bossed beam is more complex. Here, the curve of the centre deflection displays a local minimum and a local maximum, see Fig 6. This is caused by the presence of the cap. For a deformed structure, the cap end which is supported by the beam will rotate and hence both the cap and the beam will be axially strained. The larger the angle of rotation, the larger the axial stress. This leads to a larger effect of the cap on the overall stiffness of the structure, resulting in a smaller deflection. At the local minimum, the cap end coincides with an inflection point. Further increase of the cap length results in a decreasing angle of rotation of the cap end, thus a smaller contribution of the cap to the overall stiffness and hence an increase of the deflection. For the bossed beam, the angles of rotation of the cap ends are always zero. Therefore the effect of axial stiffening as described above will not occur. The steady increase of both the centre deflection and the strain for the bossed structure with increasing \( L_c \) is mainly a result of a decrease of the boss length for a given total beam length, which obviously lowers the stiffness of the structure. Due to the stiffening effect of the boss, the centre deflection and the strain are smaller for the bossed structure. The effect of the boss on the strain is small, however. For small cap lengths (<200 \( \mu m \)), the strain for the bossed structure is even slightly higher.

![Fig 6 Predicted force-induced strain and centre deflection of a bossed and non-bossed beam as a function of the cap length. The gauge has a fixed length (130 \( \mu m \)) and a fixed position, \( \varepsilon_c \), 10 \( \mu m \) from the clamped edge, and Young’s modulus \( E_b = 175 \) GPa.](image-url)
This is due to the normal strain induced in the non-bossed structure, which always results in a decrease of the totally induced strain, while for the bossed structure the normal strain is always zero. The strain for the non-bossed beam steadily increases with increasing cap length (apart from an observed local maximum for caps covering almost the entire supporting beam). At first sight, this is surprising, since the overall stiffness, as indicated by the centre deflection, displays a maximum. The reason for the steady increase of the strain is the position of the inflection points. For the non-bossed structure, the deformed shape of the supporting beam exhibits three inflection points: one underneath the cap, another one outside the cavity and the third one coinciding with the cap end. The inflection point underneath the cap is relevant for the strain induced in the gauge. For small values of $L_c$, the gauge extends over this inflection point. This will cause a relatively small difference in the angles of rotation and thus results in relatively small bending strains, see eqn (4). If $L_c$ increases, the inflection point will always move away from the gauge, thus bringing the gauge into a region with high curvature and hence resulting in an increase of the induced strain. For the bossed structure, there is only one inflection point in the region between the boss and the frame. The inflection point is always located exactly halfway between the boss boundary and the boundary of the frame. The deflection curve is antisymmetric with respect to this inflection point. For $L_c = 150 \mu m$, this implies that the inflection point is located exactly halfway along the gauge, resulting in a zero difference in the angles of rotation of the end points and thus zero induced bending strain.

Conclusions

This paper has indicated general trends and relevant design issues in order to provide a framework for developing a reliable design algorithm for micromechanical sensors utilizing encapsulated resonant strain gauges. Various design aspects have been described. To achieve an optimum performance of the resonant strain gauges, they have to be held in evacuated cavities. Highly sensitive gauges generally require a high length/thickness ratio, with a low residual (tensile) strain. Further, a high aspect ratio for the supporting structure is advantageous to achieve a large load-to-strain converting factor. The sealing cap forms an integral part of the mechanical structure and can have a strong degrading effect on the load-to-strain conversion, thereby reducing the sensitivity of the device. The influence of the cap can be minimized by decreasing its relative thickness, increasing its length, using a construction material with a small Young’s modulus, or by decreasing the gap height. A constraint is formed by the minimum stiffness required to avoid collapse caused by the ambient pressure.

The influence of the cap on the position of the neutral axis and of the inflection points can be eliminated if the cap itself is not attached to the deforming part of the structure. Bossed structures, where the cap is supported by the boss on one end and by the sensor frame or substrate on the other end, are proposed as an elegant way to achieve this, with virtually no loss of sensitivity. Also, the design complexity is greatly reduced for bossed structures, allowing for (simple) analytical models to give reliable predictions of the device performance.

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References