ON THE RELATION BETWEEN THE BASE OF AN EI ALGEBRA AND WORD GRAPHS

Xin Wang* and Cornelis Hoede

University of Twente
Department of Applied Mathematics, Faculty of EEMCS
P.O. Box 217
7500 AE Enschede
The Netherlands

Abstract

This paper is an attempt to investigate the possibilities to link algebraic fuzzy set theory with the theory of word graphs. In both theories concepts are studied and concepts can be set in correspondence. This enables to use algebraic results in the context of word graph theory.

Key words: EI algebra, graph, knowledge representation

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* On leave from Dalian Maritime University and Dalian University of Technology, Dalian, P. R. China

1. INTRODUCTION

AFS theory was proposed by Xiao Dong Liu in 1995. Much relevant research was done in recent years. AFS theory is based on AFS structures, a special kind of combinatorial objects, and AFS algebra, a family of completely distributive lattices. AFS theory has been applied to fuzzy clustering analysis, fuzzy classifiers designs, pattern recognition and hitch diagnoses, fuzzy cognitive maps, concept representations, fuzzy decision trees, fuzzy identification of systems, credit rating analysis etc.

Word graph theory was developed from 1982 on at the University of Twente. The basic idea is to describe words by graphs. The links, edges or arcs, are of a restricted number of types. The vertices represent units of perception and may correspond to both simple and very complex concepts. Word graph theory has been applied to model expert systems, decision support systems, as well as to describe texts. So word graphs also belong to the field of knowledge representation. It is this aspect that ties up knowledge graphs with AFS theory. In an earlier paper [1] the authors considered fuzzy concepts and in this paper we want to investigate the relation between AFS theory and knowledge graph theory further.

In Section 2 we will recall basic facts of AFS theory. Knowledge graphs and word graphs will be discussed in Section 3. In Section 4 we will focus on the methods of finding the basis of an EI algebra In Section 5 we will shortly discuss hypergraphs. We will show in Section 6 how word graphs can be embedded in the algebraic framework. Section 7 shortly discusses the linguistic link between the two theories.

2. EI ALGEBRA

The EI algebra is one of the AFS algebras which are a family of completely distributive lattices. It has been proven that the EI algebra has a more general algebraic structure than the Boolean algebra.

Let M be a set of fuzzy or crisp concepts, we then consider

$$EM^* = \{\sum_{i\in I} A_i \mid A_i \subseteq M,\ i \in I,\ I \text{ is any finite non-empty indexing set}\}.$$

A ‘‘standard’’ notation $\sum_{i\in I} A_i$ for the summation is $A_1 + A_2 + \ldots + A_n$. The operation + applied to two concepts, is to be interpreted as an “or”-operation, so if C_1 and C_2 are concepts, $C_1 + C_2$ is a new concept which can be understood as “C_1 or C_2 ”. A_i denotes a new concept coming from the operation of taking the union of elements of M, we will call this operation “and”. $A_1 + A_2 + \ldots + A_l$ denotes a new concept formed by the operation “or” between all the concepts A_i, $i = 1, 2, \ldots, l$, that were formed by the operation “and”.

For example, let M = {female, teacher, male, doctor, lawyer, worker}, then {female, teacher} + {male, doctor} is an element of EM^*, namely the new concept of “female teacher or male doctor” , which was formed by first the operation “and” on concepts "female" and "teacher" , respectively "male" and "doctor", and second the operation "or" on concepts "female teacher" , and "male doctor".

In [3, 4], on EM^* an equivalence relation R is defined, and we denote EM^*/R as EM. Two elements, are equivalent if

R: $(\sum_{i\in I}A_i, \sum_{j\in J}B_j)\in R \Leftrightarrow \forall i\in I, \exists j\in J$, such that $A_i \supseteq B_j$ and $\forall j\in J, \exists i\in I$ such that $B_j \supseteq A_i$.

It is easy to prove

Proposition 1: Let M be a set, $\sum_{i\in I}A_i \in EM$, if $A_u \subseteq A_v$, $u, v\in I$, $u\neq v$, then $\sum_{i\in I}A_i = \sum_{i\in I, i\neq v}A_i$, which means that $\sum_{i\in I}A_i$ and $\sum_{i\in I, i\neq v}A_i$ are equivalent.

Definition 1: For any $\sum_{i\in I}A_i, \sum_{j\in J}B_j \in EM$,

$$\sum_{i\in I}A_i \vee \sum_{j\in J}B_j = \sum_{u\in U}C_u \quad (1.1)$$

and $$\sum_{i\in I}A_i \wedge \sum_{j\in J}B_j = \sum_{i\in I, j\in J}A_i \cup B_j. \quad (1.2)$$

Here U is the disjoint union of I and J. For $u\in U$, $C_u = A_u$, if $u\in I$ and $C_u = B_u$, if $u \in J$. $(EM, \wedge, \vee)$ is called the *EI algebra* over M.

Let M be a set of fuzzy or crisp concepts, then a great number of fuzzy concepts can be expressed by the elements in EM and the fuzzy logic operations of them can be implemented by the operations $\wedge$, $\vee$ of the completely distributive lattice$(EM, \wedge, \vee)$. If attribute set M has n elements, then there are more than $\sum_{i=1}^{n}(2^{\binom{n}{i}} - 1)$ elements in EM and each element can be interpreted semantically. A few fuzzy concepts in M play a role similar to the role of a basis used in linear algebra. The logic operations of fuzzy sets in EM can be implemented by the logical operations expressed on a few fuzzy concepts in M. Let us stress that the complexity of human concepts or attributes is typically a direct result of the combinations of a few relatively simple concepts. We now consider the definitions in [2] related with the base of an EI algebra.

Definition 2: Let M be a set, EM be the EI algebra over M. $S \subseteq EM$, $(S, \wedge, \vee)$ is called a *sub-algebra* of $(EM, \wedge, \vee)$ if for any $\alpha, \beta \in S$, (1) $\alpha \vee \beta \in S$, (2) $\alpha \wedge \beta \in S$.

Definition 3: Let M be a set, EM be the EI algebra over M, $\Lambda \subseteq EM$. Then $(\Lambda)_{\mathrm{EI}} = \{ \vee_{i\in I} (\wedge_{\gamma\in T_i}\gamma) \mid T_i \subseteq \Lambda, i\in I, I$ is any indexing set$\}$ is a sub-algebra of EM. $(\Lambda)_{\mathrm{EI}}$ is called sub-algebra of *EM generated by Λ*.

Definition 4: Let M be a finite set, EM be the EI algebra of M, $D=\{\alpha_1, \alpha_2, \ldots, \alpha_n\} \subseteq EM$ $\alpha_1, \alpha_2, \ldots, \alpha_n$ are called *EI independent* if $\forall \alpha_i \in D$, $\alpha_i \notin (D\backslash\{\alpha_i\})_{\mathrm{EI}}$, otherwise $\alpha_1, \alpha_2, \ldots, \alpha_n$ are called *EI dependent*.

Definition 5: Let M be a finite set, $S \subseteq EM$, S a sub-algebra of EM, $\Lambda \subseteq S$, $\Lambda=\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, then Λ is called a base of S if 1) $(\Lambda)_{\mathrm{EI}} = S$, 2) $\alpha_1, \alpha_2, \ldots, \alpha_n$ are EI independent.

3. WORD GRAPH THEORY

In [1] the authors studied fuzzy concepts against the back ground of knowledge graph theory. In that theory concepts are represented by graphs, the vertices of which are concepts themselves. The links between the vertices are of a restricted number of types, like EQU(al) for two synonyms, ALI(ke) for similar concepts, SUB(set) for describing the (material) part-of relationship, etc. As in [1] we will not distinguish types, as our main goal is to relate such *word graph* or *concept graphs* with the algebraic structure discussed in Section 1. As an example, that we will also discuss later, we considered the concept "credit". In the appendix we give 10 definitions, as found in internet and also for each definition a *definition graph.* Combining these 10 definition graphs into one gives the concept graph for "credit".

The concept is fuzzy in that not all definitions are the same. Words occurring at least 2 times are:

v_1	reputation(5 times)	v_7	opinion(2 times)
v_2	trust (5 times)	v_8	promise(2 times)
v_3	others(3 times)	v_9	payment(2 times)
v_4	confidence(3 times)	v_{10}	goods (2 times)
v_5	person (2 times)	v_{11}	esteem(2 times)
v_6	buying(2 times)	v_{12}	honor(2 times)

Restricting the concept graph to these 12 vertices gives the disconnected graph of Figure 1.

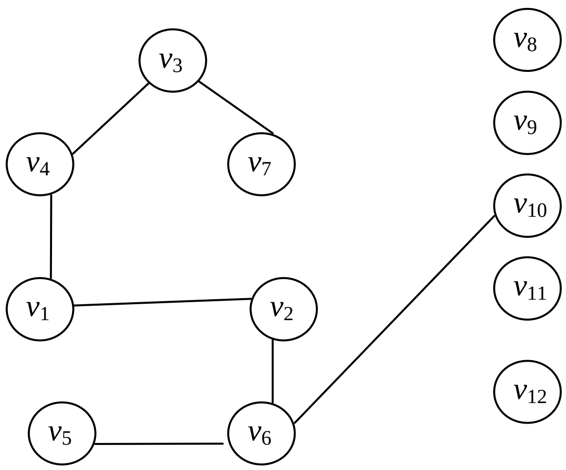

Figure 1: combined definition graph for "credit"

The rather diverse definitions yield a somewhat unsatisfactory graph. We identify v_1 and v_7, "reputation" and "opinion" and use the word v_7, opinion. We also identify v_2 and v_4, "trust" and "confidence" and use the word v_2, trust. Then we identify v_6 and v_9, "buying" and “payment” and use the word v_9, payment. Of the other concepts we only keep v_8, “promise” and relate it to v_9, payment, to obtain Figure 2.

We have added three double links, using our personal view. The trust in the promise, made by the person, is based on the opinion, that others have on the person. In Section 6 we will see that also the concepts on which the opinion is based can be included in the discussion.

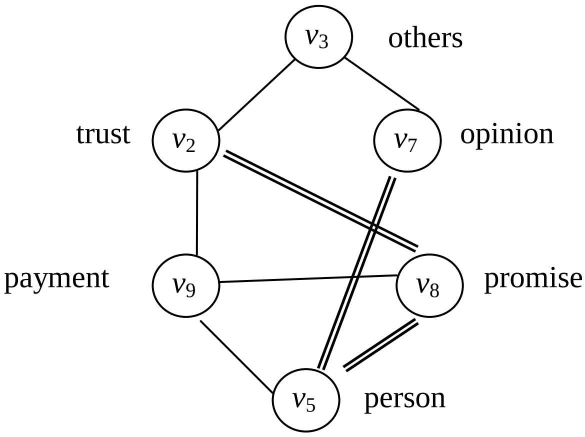

Figure 2: transformed combined definition graph

4. FINDING A BASE OF AN EI ALGEBRA

Let M be a finite set, EM be the EI algebra of M, then two lemmas and two algorithms for finding the base of a given sub-algebra of EM are the following:

Lemma 1: Any two elements generate a sub-algebra which contains at most 4 elements.

Lemma 2: The elements of EM that also belong to M must be in any base of EM.

Algorithm1: Finding a base of a sub-algebra

1: Initialize the set S to contain all the elements in EM and $B=\emptyset$.

2: If S contains more than two elements, then:

> Form the base B by selecting two elements α_1 and α_2 randomly in S, $B := \{\alpha_1, \alpha_2\}$ and apply the operations “$\wedge$, $\vee$” in EM. If some new elements α_3, α_4, which differ from these two elements and are contained in S, are generated, then update S with $S := S \setminus \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

Else

> the base $B = S$, go to End.

3: Repeat

> Select one element α_k randomly from S and add it to B, $B := B \cup \{\alpha_k\}$.
> Apply the operations of generating a sub-algebra (Definition 3) by B. If any new element α_i also belongs to S, then update S with $S: = S \setminus \{\alpha_i\}$. If any new element α_k also belongs to B, then update B with $B: = B \setminus \{\alpha_k\}$.

Until $S = \emptyset$ or $B = S$.

End.

From this algorithm we can see that it is possible that the starting elements α_1 and α_2 can eventually be deleted from the base B. This algorithm can find one base but not all the bases.

Algorithm 2a: Finding all the bases of a sub-algebra

1. Initialize the set S to contain all the elements in EM. Initialize the list L of bases as $L=\emptyset$.
2. List all the sets which contain two elements of S and put them into P: $P = \{p_1, p_2, \ldots, p_n\}$.Apply the operations "$\wedge$, $\vee$" to each pair $p_i = \{p_{i1}, p_{i2}\}$ of two elements , and add any new elements q_{i1} and q_{i2} to p_i, so $r_i = \{p_i \cup \{q_{i1}, q_{i2}\}\}$. Then form set R, $R = \{r_1, r_2, \ldots, r_n\}$.
3. Initialize $B=\emptyset$.
   If there is a set S_0 of m elements $\{\alpha_1, \alpha_2, \ldots \alpha_m\}$ in S which also belong to M, then
   $B := B\cup S_0 = S_0$, by Lemma 2. Apply the operations "$\wedge$, $\vee$" in all possible ways to all elements in B.
   If B generates EM, then
   B is a base, which, also by Lemma 2, is minimal. This base is the only base, put it into L, go to End.
   Else update P by deleting all the pairs which are generated by the elements of B.
4. For each combination of elements of P extend B with the elements of the sets r_j corresponding to the chosen elements p_j of the combination and with the elements generated by B. Apply the operations of generating a sub-algebra (Definition 3) by B. If EM is generated, then the elements originally contained in B and the elements from S potentially form a base, that can be added to L.
5. From the list L of potential bases remove those that contain another potential base in L.

End.

Algorithm 2a can find all the bases, of course including the minimum base. This algorithm is using "brute force", in particular in step 4. Before describing a more sophisticated algorithm we want to discuss the following points.

First, if all one element sets generate the algebra, then they form a base. By Lemma 2 this is a minimal base that is contained in any potential base as considered in step 4. Hence these potential bases are not bases, because the elements are not necessarily independent.

Second, it is possible there are more than one base for a sub-algebra, we are more interested in finding minimal bases than in finding all bases. This strongly suggests to consider a procedure in which bases are grown by addition of single elements. By Lemma 1 two elements generate a sub-algebra of 2 or 4 elements. In case there are no single–element sets in the algebra that have to be chosen in the base, due to the binary character of the operations $\wedge$ and $\vee$, the procedure has to start with considering P, the set of pairs. A pair p_i generates a set r_i. Addition of a third element, not in r_i, to p_i means that then we consider three elements and three pairs are considered each pair p generating a set r. The elements of the three sets r generate another sub-algebra, that might be the whole algebra, in which case the three considered elements form a base of three elements. If not, extension with a fourth element, not in any of the three sets r, can be considered leading to $\binom{4}{2}$=6 pairs p with corresponding sets r, etc.

Third, we want to point out that a graph-theoretical problem can be formulated, the solutions to which correspond with the bases we are looking for. We only consider the case in which there are no single element sets. (If there are there is no essential difficulty introduced). We represent the elements of P as the vertices of one class of a bipartite graph and the elements of the *EI* algebra as the vertices of the other class. A vertex p_i is connected to the elements in r_i. Our problem then is to find sets of vertices P, linked to all vertices of the other class, the pairs of elements of which jointly form an independent set of elements in the algebra. We will not pursue this line of reasoning, but will describe a second algorithm to find all base without using the idea of potential base.

Algorithm 2b: Finding all the bases of a sub-algebra

1. Initialize the set S to contain all the elements in *EM*. Initialize the list L of bases as $L=\emptyset$.
2. List all the sets which contain two elements of S and put them into P: $P = \{p_1, p_2, \ldots, p_n\}$.
3. Initialize $B=\emptyset$.
   If there is a set S_0 of m elements $\{\alpha_1, \alpha_2, \ldots \alpha_m\}$ in S which also belong to M, then $B := B \cup S_0 = S_0$, by Lemma 2. Apply the operations of generating a sub-algebra (Definition 3) by B.
   If B generates *EM*, then
   B is a base, which, also by Lemma 2, is minimal. This base is the only base, put it into L, go to End.
   Else
   update P by delete all the pairs which are generated by the elements of B.
4. For each element p_i in P, Do
   Put p_i into B, so $B := B \cup p_i$. Apply the operations of generating a sub-algebra (Definition 3) by B. If any new element α_k also belongs to B, then update B with $B: = B \setminus \{\alpha_k\}$.If they generate *EM*, then the elements contained in B form one base, put it into L as a new element.
   For each two elements p_i, p_j in P, Do
   Put p_i, p_j into B, so $B := B \cup p_i \cup p_j$. Apply the operations of generating a sub-algebra (Definition 3) by B. If any new element α_k also belongs to B, then update B with $B: = B \setminus \{\alpha_k\}$. If they generate *EM*, then the elements contained in B form one base, put it into L as a new element.
   :
   :
   :
   For all the pairs in P, Do

Put P into B, so $B := B \cup P$. Apply the operations of generating a sub-algebra (Definition 3) by B. If any new element α_k also belongs to B, then update B with $B: = B \setminus \{\alpha_k\}$. If they generate EM, then the elements contained in B form one base, put it into L as a new element.

End.

5. HYPERGRAPHS

Definition 6: A *hypergraph* $H = (V, \mathcal{E})$ consists of a set V of vertices and a set $\mathcal{E}$ of **hyperedges,** nonempty subsets of V. [5]

A graph G = (V, E) consists of a set of vertices and a set E of pairs of vertices, called edges. A hyperedge can be any nonempty subset of V, so also a single vertex can be a hyperedge. Hypergraphs are also called *set systems*. Comparing V with the set M of an EI algebra, the subsets of M can be seen as hyperedges of a hypergraph with M as vertex set.

We want to establish a map between the elements of the EI algebra and hypergraphs. Let M be a finite set, and let $(EM, \wedge, \vee)$ be the EI algebra of M. The elements of EM can be cast in a standard form using only the $\vee$ operation. We map the elements $\alpha = \sum_{i\in I} A_i$ of EM to hypergraph $H(M, \mathcal{E})$, and each A_i is corresponding to one hyperedge.

Example 1: $M = \{1, 2, 3, 4\}$, $\alpha = A_1 + A_2 = \{1, 2, 4\}+\{2, 3\} \in EM$, $\mathcal{E} = \{\{1, 2, 4\}, \{2, 3\}\}$, then the element α in EM can be mapped to the hypergraph $H(M, \mathcal{E})$. $A_1 = \{1, 2, 4\}$ is corresponding with hyperedge {1, 2, 4} and A_2 with {2, 3}.

In analogy with the operations in the EI algebra we formally define the following two operations on hypergraphs.

Definition 7: Let $H_i(V, \mathcal{E}_i)$ and $H_j(V, \mathcal{E}_j)$ be hypergraphs, then

$$H_i(V, \mathcal{E}_i) \wedge H_j(V, \mathcal{E}_j) = H(V, \mathcal{E}_p),$$

where $\mathcal{E}_p = \{e \mid e = e_i \cup e_j, \forall e_i \in \mathcal{E}_i, \forall e_j \in \mathcal{E}_j\}$ and

$$H_i(V, \mathcal{E}_i) \vee H_j(V, \mathcal{E}_j) = H(V, \mathcal{E}_k),$$

where $\mathcal{E}_k = \mathcal{E}_i \cup \mathcal{E}_j$.

Corresponding with Proposition 1, we give the Proposition 2:

Proposition 2: Let $H(V, \mathcal{E})$ be a hypergraph, $\varepsilon_1, \varepsilon_2 \in \mathcal{E}$, if $\varepsilon_1 \subseteq \varepsilon_2$ then

$$H(V, \mathcal{E}) = H(V, \mathcal{E}'), \text{ where } \mathcal{E}' = \mathcal{E} \setminus \varepsilon_2.$$

Example 2: $\mathcal{E} = \{\{1, 2, 4\}, \{2, 3\}\}$, $\mathcal{E}' = \{\{3\}\}$, then $H(V, \mathcal{E}) \vee H(V, \mathcal{E}') = H(V, \mathcal{E}'')$, where $\mathcal{E}'' = \{\{1, 2, 4\}, \{2, 3\}, \{3\}\} = \{\{1, 2, 4\}, \{3\}\}$, by proposition 2.

$H(V, \mathcal{E}) \wedge H(V, \mathcal{E}') = H(V, \mathcal{E}''')$, where $\mathcal{E}''' = \{\{1, 2, 3, 4\}, \{2, 3\}\} = \{\{2, 3\}\}$, also by proposition 2.

In this way the operations on the EI algebra are simply translated to operations on hypergraphs.

6. RELATING EI ALGEBRAS WITH WORD GRAPHS

Having established a direct correspondence between EI algebras and hypergraphs, we now describe how word graphs can be represented as hyperedges of a hypergraph.

Recall that M was seen as a set of attributes. Now, let us look at another example. Let M be a set of some simple attributes. For all $\alpha = \sum_{i\in I} A_i \in EM$, α has a well-defined semantics. For example, suppose that the simple attributes in M are as follows: m_1 = male, m_2 = female, m_3 = salary high, m_4 = old, m_5 = high fortune. Let $\alpha = \{m_1, m_3, m_4\} + \{m_1, m_4, m_5\} + \{m_2, m_3\} + \{m_2, m_5\} \in EM$. α can be considered as a description of a complex attribute ‘‘credit’’ (interestingly enough, this complex concept of ‘‘credit’’ implies that there could be different individuals coming with different combinations of the simple attributes). This attribute is what we referred to in Section 3 as basis for the opinion.

We could see the attribute set M as the vertex set of a graph without edges. M is a concept set, but a word graph is a more general structure as vertices, the concepts, are also linked by an edge set E. Let $G(V, E)$ be the word graph, then we interpret both V and E as concepts and consider

$$M = V \cup E$$

as the hyperedge of a hypergraph. Note that in $H(V, \mathcal{E})$, $\mathcal{E}$ may contain the hyperedge V. This is the situation that we are considering for the word graph. Any subgraph of the word graph G is also a subset of M and therefore also a hyperedge on the basic set M, see the appendix.

The operations $\wedge$ and $\vee$ on elements of the EI algebra have corresponding operations on hypergraphs.

Let us now consider a combined definition graph of a concept. Each of the definition (word) graph of a concept used in its construction can be seen as a hyperedge on the set M of vertices and edges of the combined definition graph. The EI algebra generated by these subsets of M has elements that are corresponding to graphs or to pairs, triples, etc. of graphs. A minimal base of the algebra determines a set of concepts, word graphs.These word graphs correspond to words that form a natural ontology for describing the combined definition. This also holds in case for a certain knowledge graph, describing a certain field, one or more ontologies, sets of words, are used. The described translation to an EI algebra enables to find a most appropriate ontology.

7.DISCUSSION

The AFS framework provides an effective tool to convert the information in training examples and databases into membership functions and their logical operations, and the membership functions and their logical operations are determined by algorithms based on the distribution of the original data.

Theoretical studies and their applications show that the AFS framework is a new approach to knowledge representation and inference that is essential to any intelligent system. It offers a flexible and powerful framework for representing human knowledge and studying large-scale intelligent systems in real world applications.

In this paper we are not so much interested in the fuzzy aspects. Of course, the concept “credit” is a fuzzy concept in the sense of our earlier paper[1]. The frequencies with which vertices and edges occur in the definition graphs determine a natural membership degree with respect to the basic set *M*. Every word graph and its corresponding hyperedge, i.e. set of vertices and edges, has a natural membership function in this way.

Our interest lies primarily with the remark in the paper of Wang and Liu[2], that “by this approach AFS theory can be used to study natural language”. In Section 6 we already mentioned the example $\alpha = \{m_1, m_3, m_4\} + \{m_1, m_4, m_5\} + \{m_2, m_3\} + \{m_2, m_5\} \in EM$, that could be considered as a description of a complex attribute “credit”. It is an element of an EI algebra on the set *M* of attributes. A graph theoretical interpretation of a set of attributes is a graph consisting only of vertices. We quote: “α means the concept: man who is old and whose salary is high or a man who is old and with high fortune or a woman whose salary is high or a woman with high fortune”.

The attributes man, woman, high salary and high fortune do not occur in our combined definition graph for “credit”. It is clear that there is a difference in the interpretation of the concept. Each of the four combinations of the four attributes is considered to “define” credit. But “woman with high salary” for example, is **not** what is meant with “credit”. What is clearly meant in [2] is that credit can be given to a woman with high salary. A high salary is a basis for the trust that one can have in payment. A more precise analysis should be given to incorporate this into the knowledge graph of credit.

This can be done by “expanding” the concepts occurring in the combined definition graph. This means that, for example, the vertex with name trust is replaced by a word graph of “trust”. One of the links may be to a concept that has a causal relationship with trust. If this concept is salary, the expansion brings “salary” into the expanded word graph of “credit”, and, via the hyperedge, into the realm of the EI algebra. There is a “pars pro toto” construction. A part of the knowledge graph for “credit” is used to talk about “credit”.

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APPENDIX

The 10 definitions of "credit" found are the following:

1. A reputation for sound character or quality; standing.

2. Influence based on the good opinion or confidence of others.

3. Reputation for solvency and integrity entitling a person to be trusted in buying or borrowing.

4. To confide in the truth of; to give credence to; to put trust in; to believe.

5. Reliance on the truth of something said or done; belief; faith; trust; confidence.

6. Reputation derived from the confidence of others; esteem; honor; good name; estimation.

7. That which tends to procure, or add to, reputation or esteem; an honor.

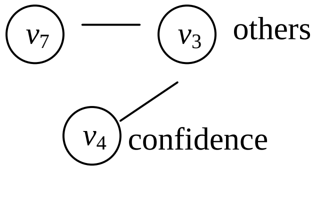
8. Influence derived from the good opinion, confidence, or favor of others; interest.

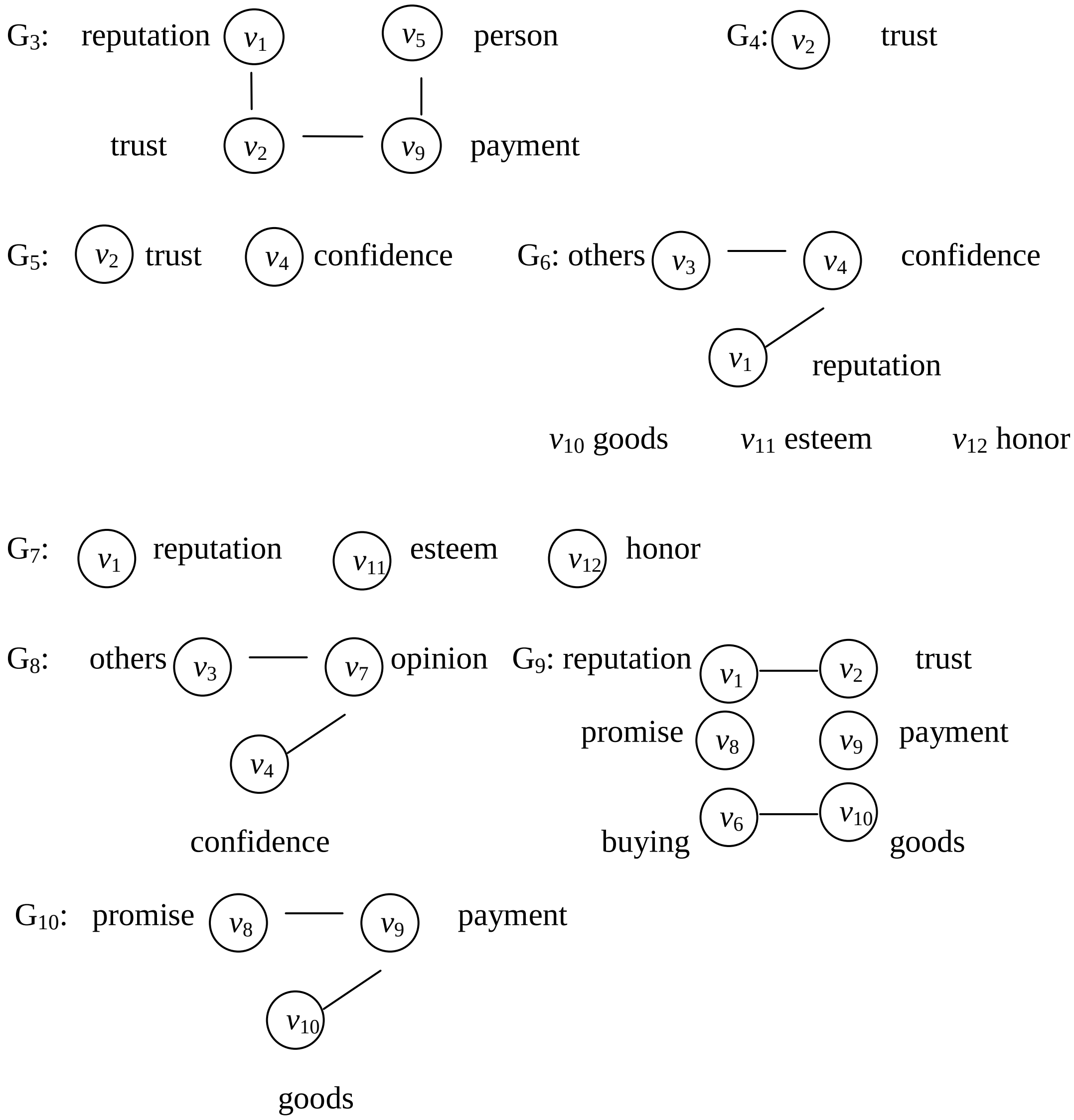
9. Trust given or received; expectation of future payment for property transferred, or of fulfillment or promises given; mercantile reputation entitling one to be trusted; applied to individuals, corporations, communities, or nations; as, to buy goods on credit.

10. Credit refers to the ability to obtain money, goods or services in the present against the promise to pay for them in the future.

We now list the 10 graphs, restricted to the 12 concepts mentioned in Section 2:

G_1: 

G_2: opinion 



These definition graphs can also be seen as hyperedges. The vertex set of the hypergraph $H = (M, \mathcal{E})$ is :

$M = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, \{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_9\}, \{v_3, v_4\}, \{v_3, v_7\}, \{v_4, v_7\}, \{v_5, v_9\}, \{v_6, v_{10}\}, \{v_8, v_9\}, \{v_9, v_{10}\}\}$.

The hyperedges in $\mathcal{E}$ are:
$\mathcal{E}_1 = \{v_1\}$, $\mathcal{E}_2 = \{v_3, v_4, v_7, \{v_3, v_4\}, \{v_3, v_7\}\}$, $\mathcal{E}_3 = \{v_1, v_2, v_5, v_9, \{v_1, v_2\}, \{v_2, v_9\}, \{v_5, v_9\}\}$, $\mathcal{E}_4 = \{v_2\}$, $\mathcal{E}_5 = \{v_2, v_4\}$, $\mathcal{E}_6 = \{v_1, v_3, v_4, v_{10}, v_{11}, v_{12}, \{v_1, v_4\}, \{v_3, v_4\}\}$, $\mathcal{E}_7 = \{v_1, v_{11}, v_{12}\}$, $\mathcal{E}_8 = \{v_3, v_4, v_7, \{v_3, v_7\}, \{v_4, v_7\}\}$, $\mathcal{E}_9 = \{v_1, v_2, v_6, v_8, v_9, v_{10}, \{v_1, v_2\}, \{v_6, v_{10}\}\}$, $\mathcal{E}_{10} = \{v_8, v_9, v_{10}, \{v_8, v_9\}, \{v_9, v_{10}\}\}$.