Variable-Voltage Class-E Power Amplifiers

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Abstract—The Class-E power amplifier is widely used due to its high efficiency, resulting from switching at zero voltage and zero slope of the switch voltage. In this paper, we extend general analytical solutions for the Class-E power amplifier to the ideal single-ended Variable-Voltage Class-E (Class-E_{VV}) power amplifier that switches at zero slope but not necessarily at zero voltage.

The theory is verified by simulations and measurements, and show that the peak switch voltage of Class-E_{VV} power amplifier is lower (up to ≈ 30%) than in the conventional Class-E power amplifier; which can be utilized to obtain e.g. higher output power (up to ≈ 200% more) with lower power-added efficiency (maximum ≈ 20% less) from technologies with low-breakdown voltages.

Index Terms—Power amplifiers, switching amplifiers.

I. INTRODUCTION

The Class-E power amplifier (PA) can achieve high efficiency due to its tuned load network that shapes the switch voltage to zero-voltage and zero-slope at the switch turn-on moment. Many different aspects of the Class-E power amplifier (PA) have been extensively studied in the last three decades [1]-[7]. The reportedly “sub-optimum operation” with nonzero-voltage or nonzero-slope switching [3] is hardly used because of its believed inferior performance.

![Figure 1](image)

Fig. 1. Sub-optimum operation classes of Class-E PA

In [1], zero-voltage and nonzero-slope switching operation only for RF-choke Class-E PAs have been investigated and it is found that these Class-E PAs have about 10% higher tolerance to switch (transistor) output capacitance than conventional RF-choke Class-E PAs. In [6], it has been shown that zero-voltage and nonzero-slope switching (denoted as variable-slope Class-E, Class-E_{VS} (Fig.1)) operation of finite dc-feed inductance Class-E PAs allows using significantly larger switch (transistor) size (up to 110% more); which can be utilized in obtaining higher efficiency.

To the best of the authors’ knowledge, analyses of nonzero-voltage and zero-slope switching Class-E PAs (denoted as Variable-Voltage Class-E, Class-E_{VV} (Fig.1)) have not been presented in literature. This paper presents an analysis and some discussions on Class-E_{VV} PAs to utilize their lower peak switch voltage feature to obtain higher output power with reasonably high power-added efficiency from transistor technologies with low-breakdown voltages. The analysis in this paper is based on closed form expressions like those presented in [5], [6] and [7] for Class-E PAs\(^1\) In the analysis, the finite dc-feed inductance \(L\), the switch (transistor) input/output capacitance \((C_{in}/C_{out})\) and on-resistance \((R_{on})\) (Fig. 2b) are all taken into account. As a result of the analysis, analytical design equations are presented showing the relation between the input parameters and the circuit component values. It is shown in this paper that Class-E_{VV} can have up to ≈ 30% lower peak switch voltage than conventional Class-E PAs; which can be used to obtain up to ≈ 200% more output power with maximum ≈ 20% lower power-added efficiency\(^2\) assuming the same transistor size, reliable peak voltage, matching network and pre-driver.

![Figure 2](image)

Fig. 2. (a) Class-E_{VV} PA including pre-driver and matching network (b) Model of Class-E_{VV} PA (c) Normalized switch voltage and switch current of Class-E_{VV} PA with turn-on voltage of \(\alpha = 2\), \(\alpha = 0\) (Class-E) and \(\alpha = 1\)

\(^1\)In [5] and [7] only Class-E operation and in [6] only Class-E_{VS} operation is considered.

\(^2\)Power is lost due to discharging of the capacitor \((C)\) to ground via the switch at the switch turn-on moment.
II. ANALYSIS OF CLASS-E\textsubscript{\textsc{V}}\textsc{V} POWER AMPLIFIER

A single ended switching PA topology and its model are given in Fig.2a and Fig.2b respectively. An analytical solution for the model in Fig.2b to operate as a Class-E PA (e.g. switching at zero-voltage and zero-slope) is given in [5] and [7]\textsuperscript{3}. In [6], switching with variable slope aspect was generalized and analytical design equations for Class-E\textsubscript{\textsc{V}}\textsc{S} (e.g. switching at zero-voltage and variable-slope) is given. In the current paper, the variable voltage aspect is generalized and general analytical solution for Class-E\textsubscript{\textsc{V}} (e.g. switching at variable-voltage and zero-slope) is given based on the model in Fig. 2b. If the correct input parameters and circuit element values are chosen, the circuit in Fig. 2a properly operates as a Class-E\textsubscript{\textsc{V}}\textsc{V} PA by satisfying the following conditions (1):

\[ v_C(2\pi/\omega) = \alpha V_{DD} \quad \text{and} \quad \left. \frac{dv_C(t)}{dt} \right|_{t=2\pi/\omega} = 0 \]  

where \( \alpha V_{DD} \) is the voltage of \( v_C(t) \) at the moment the switch is closed; for conventional Class-E operation \( \alpha = 0 \). However, in Class-E\textsubscript{\textsc{V}}\textsc{V} operation \( \alpha \) is a real value\textsuperscript{4} that can be selected freely and therefore gives a degree of freedom in the design of a Class-E\textsubscript{\textsc{V}}\textsc{V} PA. A design set \( K = \{K_L, K_C, K_P, K_X, K_R\} \) (Table 1) can be derived that relates circuit element values and operating conditions such as supply voltage, operating frequency and output power for the switching PA in Fig.2a.

In this section, closed form analytical expressions for each element of the Class-E\textsubscript{\textsc{V}}\textsc{V} are derived; which enables infinitely many Class-E\textsubscript{\textsc{V}}\textsc{V} realizations, to be selected by the parameters \( q = 1/(\omega\sqrt{LC}) \), \( m = \omega R_{on} C \) [5], [6], [7]) and \( \alpha \). The design parameters \( q \) and \( m \) are free variables like \( \alpha \) and mathematically can be assigned any positive real value\textsuperscript{5}.

Table 1: Design Set \( K \) for Class-E\textsubscript{\textsc{V}}\textsc{V} PA\textsuperscript{6}

\begin{tabular}{|c|c|c|c|c|}
\hline
\( K_L = \frac{\alpha L}{C} \) & \( K_C = \omega CR \) & \( K_P = \frac{\alpha VT_m}{V_{DD}} \) & \( K_X = \frac{1}{\pi m} \) & \( K_R = \frac{\pi m}{\omega} \) \\
\hline
\end{tabular}

This section presents the derivation of the design set \( K \) for Class-E\textsubscript{\textsc{V}}\textsc{V} PAs.

A. Circuit Description and Assumptions

The circuit model of the Class-E PA is given in Fig.2b. For the analysis and the derivations in this paper a number of assumptions are made:

- the only real power loss occurs on \( R_L, R_{on}, R_{di}, R_0 \) and \( R_m \).
- The capacitors \( C_{in} \) and \( C_{out} \) are assumed to be linear.
- The switch (transistor) operates instantly with on-resistance (\( R_{on} \)) and infinite off-resistance

- the loaded quality factor (Q\textsubscript{L}) of the series resonant circuit (\( L_0 \) and \( C_0 \)) is high enough in order for the output current to be sinusoidal at the switching frequency
- the duty cycle is 50%

Fig.2c shows the switching behavior and the switch definition used: in the time interval \( 0 \leq t < \pi/\omega \) the switch is closed and in \( \pi/\omega \leq t < 2\pi/\omega \) it is opened. This switching repeats itself with a period of \( 2\pi/\omega \).

B. Circuit Analysis

In the analysis, the current into the load, \( i_R(t) \), is assumed to be sinusoidal. Note that theoretically this occurs only for infinite Q\textsubscript{L} of the series resonant network consisting of \( L_0 \) and \( C_0 \). It is however a widely used assumption [1], [3], [5] that simplifies analysis considerably: \( i_R(t) = I_R\sin(\omega t + \varphi) \).

In the time interval \( 0 < t < \pi/\omega \), the switch is closed. The KCL at the drain node can be written as:

\[ i_L(t) - i_S(t) - i_C(t) + i_R(t) = 0 \]  

Relation (2) can be arranged in the form of a linear, non-homogenous, second order differential equation

\[ C^2\frac{dv_{con}(t)}{dt^2} + \frac{1}{R_{on}} \frac{dv_{con}}{dt} - \frac{V_{DD} - v_{con}}{L} - \omega I_R \cos(\omega t + \varphi) = 0 \]  

which has as solution

\[ v_{con} = \frac{q^2 \sin(\omega t + \varphi) - \left( q^2 - q^4 \right) \cos(\omega t + \varphi)}{1 + (m^2 + 1) q^2 - \left( 2q^2 + \frac{pV_{DD}}{V_{DD}} \right) + e^{\omega t} C_{on1} \omega C_{on2} + e^{\omega t} C_{on1} \omega C_{on2} + e^{\omega t} C_{on1} \omega C_{on2}} \]  

where, \( a = -1 + \sqrt{1 - 4q^2m^2} \), \( b = -1 - \sqrt{1 - 4q^2m^2} \) and \( p = \frac{Lm}{2V_{DD}} \). \( C_{on1} \) and \( C_{on2} \) follow from the continuity of the capacitor voltage (C) and the inductor (L) current at the switch-on moment.

In the time interval \( \pi/\omega \leq t < 2\pi/\omega \), the switch is opened. Then, in the Class-E\textsubscript{\textsc{V}}\textsc{V} PA the current through capacitor C is

\[ i_C(t) = \frac{1}{L} \int_{\pi/\omega}^{t} (V_{DD} - v_{Coff}(t)) \, dt + i_L \left( \frac{\pi}{\omega} \right) + i_R(t) \]  

Relation (5) can be re-arranged in the form of a linear, non-homogenous, second-order differential equation

\[ LC \frac{d^2v_{c_{off}}(t)}{dt^2} + v_{c_{off}}(t) - V_{DD} - \omega L i_R \cos(\omega t + \varphi) = 0 \]  

which has as solution

\[ v_{c_{off}}(t) = C_{off1} \cos(q\omega t) + C_{off2} \sin(q\omega t) + V_{DD} - \frac{q^2}{1 - q^2} pV_{DD} \cos(\omega t + \varphi) \]  

\( C_{off1} \) and \( C_{off2} \) follow from the Class-E\textsubscript{\textsc{V}}\textsc{V} conditions (1).

It follows from (4) and (7) that \( v_{con}(t) \) and \( v_{c_{off}}(t) \) can be expressed in terms of \( V_{DD} \) and \( \omega \) hence be solved analytically only if \( \varphi, q, p, m \) and \( \alpha \) are known. The derivation of the four parameters \( \varphi, q, p, m \) and \( \alpha \) is the next step in the solution.

\textsuperscript{3}Note that the analysis in [5] is extended in [7] by taking into account a non-zero switch on-resistance.

\textsuperscript{4}Theoretically, \( \alpha < 0 \) is possible however, for MOS type of switches the junction diodes get forward biased when \( \alpha < 0 \); decreasing efficiency.

\textsuperscript{5}Although mathematically \( q, m \) can be assigned any positive real value and \( \alpha \) can be assigned any real value, as it is shown (later) that only certain ranges results in design equations that are feasible in practice.

\textsuperscript{6}\( L_0 \) and \( C_0 \) seen in Fig. 2a can be determined from the chosen loaded quality factor (Q\textsubscript{L} = \( \omega_0 L_0/R \)) where \( \omega_0 = 1/\sqrt{L_0C_0} \).
By using the continuity of the inductor current and the capacitor voltage at the switch turn-off moment two independent equations follow that have the same format:

\[ f_i(p, q, \varphi, m, \alpha) = p a_i(q, m, \alpha) \cos(\varphi) + p b_i(q, m, \alpha) \sin(\varphi) + c_i(q, m, \alpha) = 0, \text{ where } i = 1, 2. \]

The variables \( p \) and \( \varphi \) can be solved by using \( f_1(p, q, \varphi, m, \alpha) \) and \( f_2(p, q, \varphi, m, \alpha) \) in terms of \( q, m \) and \( \alpha \) as given in the appendix. Here, \( q, m \) and \( \alpha \) are free variables that can mathematically take any positive real value.

C. Design sets for Class-E \( V_{VV} \) operation

The results of the mathematical derivation of the solutions leading to Class-E \( V_{VV} \) operation can be used to derive an easy-to-use design procedure for Class-E \( V_{VV} \) PAs. Using the result of the derivation for \( p(q, m, \alpha) \) and \( \varphi(q, m, \alpha) \), analytical expressions for the design set \( K = \{ K_L, K_C, K_P, K_X, K_R \} \) can readily be derived.

\[ K_L: \text{ follows from the principle of power conservation:} \]

\[ I_R^2 R/2 + P_{\text{switch}} = I_0 V_{DD} \quad (8) \]

In this relation, \( I_0 \) is the average supply current:

\[ I_0 = \frac{\omega}{2 \pi R_{on}} \int_0^{\pi/\omega} v_{c_{on}}(t) dt \]

and \( P_{\text{switch}} \) is the power spent on \( R_{on} \):

\[ P_{\text{switch}} = \frac{\omega}{2 \pi R_{on}} \int_0^{\pi} v_{c_{on}}(t)^2 dt \]

Substitution of (9) and \( p \) in (8) yields

\[ K_L(q, m, \alpha) = \frac{-(p V_{DD} q)^2 m \pi}{\omega \int_{\pi}^{\pi} (v_{c_{on}}(t))^2 - V_{DD} v_{c_{on}}(t) dt} \]

Since \( p \) and \( \varphi \) are all functions of \( q, m \) and \( \alpha \), \( K_L \) is a function of only \( q, m \) and \( \alpha \).

\[ K_C: \text{ follows directly from the definition of } q \text{ and } K_L: \]

\[ K_C(q, m, \alpha) = 1/(q^2 K_L(q, m, \alpha)) \]

\[ K_P: \text{ can easily be found as a function of } q, m \text{ and } \alpha \text{ by using}: \]

\[ K_P(q, m, \alpha) = p(q, m, \alpha)^2/\{2 K_L(q, m, \alpha)^2\} \]

\[ K_X: \text{ can be derived using two fundamental quadrature Fourier components of } v_{c_{on}}(t). \]

\[ v_R = \int_0^{\pi} v_{c_{on}}(t) \sin(\omega t + \varphi) dt + \int_0^{\pi} \sqrt{2} v_{c_{eff}}(t) \sin(\omega t + \varphi) dt \]

\[ v_X = \int_0^{\pi} v_{c_{on}}(t) \cos(\omega t + \varphi) dt + \int_0^{\pi} \sqrt{2} v_{c_{eff}}(t) \cos(\omega t + \varphi) dt \]

\[ K_X(q, m, \alpha) = v_X/v_R \]

\[ K_R: \text{ follows from the definition of } m \text{ and } K_C: \]

\[ K_R(q, m, \alpha) = K_C(q, m, \alpha)/m \]

D. Efficiency and Output Power of Class-E \( V_{VV} \)

Taking the three loss mechanisms \( (P_{in}, P_{\text{switch}}, P_{\text{match}}) \) shown in Fig. 2 and the existence of certain switch (transistor) breakdown-voltage\(^7\) \( (V_{BD} = z(q, m, \alpha) \cdot V_{DD}) \) into account the efficiency and the power on the antenna can be expressed:

Drain Efficiency\( (\eta) \): derived as a function of \( q, m \) and \( \alpha \).

\[ \eta(q, m, \alpha) = 1 - \frac{P_{\text{switch}}}{V_{DD} I_0} = 1 - \frac{\int_{\pi}^{\pi} v_{c_{on}}(t)^2 dt}{V_{DD} \int_{\pi}^{\pi} v_{c_{on}}(t) dt} \]

Power-Added Efficiency\( (PAE) \): as a function of \( q, m \) and \( \alpha \).

\[ PAE(q, m, \alpha) = \frac{P_{\text{antenna}} - P_{\text{in}}}{V_{DD} I_0} = \eta(q, m, \alpha) \left( \frac{1}{1 + \sqrt{\frac{n - Q_L}{Q_m}}} - \frac{c_1 c_2 K_C(q, m, \alpha)}{2d^2 Q_d K_P(q, m, \alpha)} \right) \]

where, \( P_{\text{antenna}} = P_{\text{OUT}} - P_{\text{match}}, n = \frac{R_0}{R_{on}}, P_{in} \approx \omega C_m V_{DD}\text{drive} \text{/(}2Q_d\text{)}, c_1 = C_m/C_{out}, c_2 = C_{out}/C, d = V_{DD}/V_{DD}\text{drive}, Q_m \text{ is the quality factor of the inductors } L_m \text{ and } L_0, Q_d \text{ is the quality factor of } L_d \text{ (Fig. 2b)}. \)

Power on Antenna\( (P_{\text{antenna}}) \): as a function of \( q, m \) and \( \alpha \).

\[ P_{\text{antenna}}(q, m, \alpha) = K_P(q, m, \alpha) \frac{\sqrt{2} V_{BD}^2}{z(q, m, \alpha)^2} 1 + \sqrt{\frac{n - Q_L}{Q_m}} \frac{V_{BD}}{R_L} \]

Fig. 3. PAE and \( P_{\text{antenna}} \) using technology and design parameters in [4]

The value of \( c_1, d \) and \( V_{BD} \) depend on the characteristics of the transistor technology. For a certain operation frequency and transistor technology, \( m \) only depends on the value of \( c_2 \) since \( m = \omega/\beta/c_2 \) where \( \beta = R_{on}C_{out} \) which depends on the transistor technology. Therefore, for a given transistor technology, impedance transformation ratio \( n \), \( Q_m \) and \( Q_d \), PAE and \( P_{\text{antenna}} \) both are a function of only \( q, m \) and \( \alpha \). In Fig. 3, PAE and \( P_{\text{antenna}} \) are plotted as a function of \( \alpha \) for a few values of \( q \) using the design and technology parameters in [4]. In [4][8], \( Q_d = 3, Q_\text{d} = 3, n = 3, c_2 = 1, \omega = 2 \pi \cdot 1.7 \cdot 10^9 \text{ rad/sec} \) and the transistor technology is 0.13um CMOS (thick oxide) for which \( V_{BD} \approx 2.5 \cdot 3.56 \text{ V}^9, c_1 \approx 4 \text{ and } R_{on} C_{out} \approx 10^{-12}; \text{ resulting in } m \approx 0.011. \)

\(^7\)Here \( V_{DD} \) refers to gate-drain oxide breakdown voltage which is assumed to be lower than junction breakdown voltages. It is also assumed that \( V_{DD}\text{drive} \) is chosen as max. reliable gate-source oxide breakdown voltage.

\(^8\)Value of \( q \) is calculated as 1.2 from the circuit element values in [4] and \( Q_m \) for \( L_m \) and \( L_0 \text{ (bondwire)} \) is assumed to be 30.

\(^9\)In [4], breakdown voltage is doubled by using a cascode topology.
In [4], an optimization procedure based upon approximations and simulation results is given for Class-E PAs. The PAE measurement results given in [4] (67%), for conventional Class-E PA (q = 1.2, α = 0). The difference can be attributed to the losses due to ground bonding and dc-feed inductance; which are not taken into account in the analytical design equations in this paper.

Table 2 and Fig. 4, which verify that Class-E PA can have lower peak switch voltage; which can be utilized to obtain high output power using low-voltage transistor technologies (e.g. CMOS). This paper shows (theoretically and experimentally) that Class-E PA can have up to ≈ 200% more output power than conventional Class-E PAs under the same drive, load and reliable peak voltage conditions, with only a modest PAE penalty.

REFERENCES


APPENDIX I

\[ p = \sqrt{(bgh_1 - bgh_2)^2 + (gh_3 - bgh_2)^2} \]
\[ g_1 = \frac{-e^{\alpha (A + lmq)} - e^{\alpha (A + emq)}}{B (-b+a)} \]
\[ g_2 = \frac{\cos(q\alpha) + 1}{q^2} + \frac{m^2q^2(q-1)(q+1)}{B} - \frac{e^{\alpha (A - m^2q)} + e^{\alpha (A - m^2q)}}{B (-b+a)} \]
\[ h_1 = \frac{m^2q^2}{B} + \frac{q^2 (m \sin(q) + \cos(q) - 1)}{B} - \frac{e^{\alpha (A - m^2q)} + e^{\alpha (A - m^2q)}}{B (-b+a)} \]
\[ h_2 = \frac{1}{(1-a)B (\cos(q) + m \sin(q) - 1)} + \frac{m (\cos(q) + \sin(q) + m)}{B (-b+a)} - \frac{e^{\alpha (A - m^2q)} + e^{\alpha (A - m^2q)}}{B (-b+a)} \]
\[ h_3 = \frac{m^2q^2(q-1)(q+1)}{B} - \frac{q (m \cos(q) + \sin(q) + m) + \text{max} e^{\alpha (A + bmg^2)}}{B (-b+a)} - \frac{m (\cos(q) - q)^2}{B} + \frac{m^2 e^{\alpha (A + bmg^2)}}{B (-b+a)} \]

\[ \varphi = \arctan(bgh_3 - bgh_2, gh_3 - bgh_2) \]

\[ \phi = \arctan(bgh_3 - bgh_2, gh_3 - bgh_2) \]

p and \( \varphi \) are expressed in terms of \( q, m \) and \( \alpha \) as follows:

\[ \phi = \arctan(bgh_3 - bgh_2, gh_3 - bgh_2) \]

\[ \phi = \arctan(bgh_3 - bgh_2, gh_3 - bgh_2) \]

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